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ON THE "TRIPLE MELTING" OF TRIPALMITIN

BY

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ABSTRACT

The mechanism of the "triple melting" was investigated on tripalmitin. Thermal and dielectric measurements, structure analysis by X-ray and electron diffraction methods, and microscopic observation were done for all polymorphic forms. The temperature dependency of these physical properties was examined and the behaviour of the phase transformation was observed. Some information on the stability of each phase was obtained.

The discussion on the mechanism of "triple melting" was made considering many experimental results.

1. Introduction

Triglycerides have at least three polymorphic forms. The three solid modifications have their respective melting points and there are no definite transition points among them, namely, the modification having the highest melting point is stable at all temperatures below its melting point, while the other two modifications can be obtained only as metastable phases. At temperatures below the lowest melting point any one of these three modifications can occur according to the cooling velocity from the melt, but crystallization from a solution always produces the stable modification. The so-called "multiple melting" is often observed when a sample settled in one of the metastable phases is heated at a proper velocity. According to Clarkson and Malkin (**1**), all the even and odd members of the homologous series from tricaprin to tristearin exhibit the polymorphism of this type. Since such behaviour is characteristic to triglycerides, it may be considered that it is due to the complicated shape of the triglyceride molecule which consists of three branches and is not linear.

Interesting properties of these substances as dielectrics which suggest the flexibility of the links between the alkyl groups and glycerol group have also been reported (**2**). Our work was attempted to understand the character of the triple melting, and polymorphism of tripalmitin was further studied by means of thermal analysis, dielectric measurement, microscopic observation, electron diffraction and X-ray diffraction. For

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the convenience of facilitating thermal treatment, tripalmitin was chosen for our experiments.

2. Experimental results

Tripalmitin used was prepared by reaction of purified glycerol (b.p. 169~170°C/15 mm. D^{14}·1.264) and purified palmitic acid (m.p. 60.4°C) following C. Kailan and R. Obogi's direction (**3**). It was purified by successive recrystallization from purified ethyl alcohol and pure toluene and dried in desiccator under a reduced pressure. Its melting point was 65.5°C.

To begin with, the melting points of the metastable forms were examined by thermal analysis. When the metastable form was studied, heating and cooling had to be suitably quickened because of their tendency to change into more stable form. Therefore the size of the sample had to be small in order that the uniformity of temperature within the sample might be maintained during the observation. So, a small drop of the molten substance less than 1 mg in weight was put on the junction of the thermocouple which was made of fine wires of copper and constantan 0.01~0.05 mm in diameter, and was fixed as shown in Fig. 1. Measurements were done within the temperature range between 0°C and 80°C. Fig. 2 shows two typical curves with

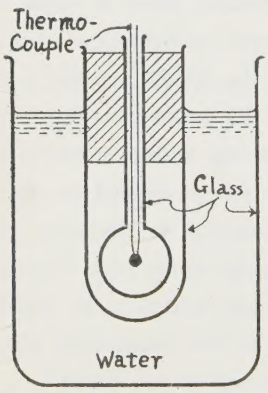

Fig. 1. A diagram for the thermal analysis.

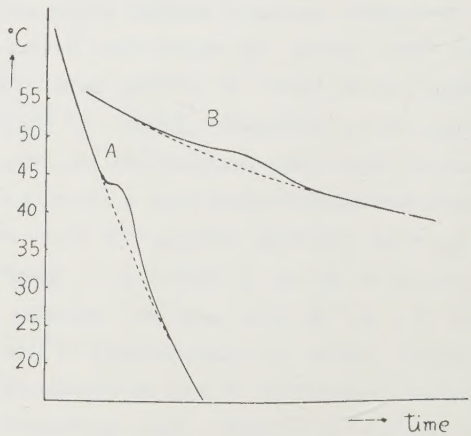

Fig. 2. Cooling curves
(A) Rapid cooling, (B) Slow cooling.

different cooling rates. The curve for rapid cooling has a hump or a peak the top of which is lower than 44°C. If the cooling is handled more slowly, the temperature at which the anomaly in cooling curve occurs, shifts upwards and at the same time its shape becomes flat as represented by curve B. The samples solidified through these processes are semi-transparent.

The heating curves A and B in Fig. 3 were obtained for the samples which had

been subjected to rapid and slow cooling respectively from the melt and they may be regarded as the continuations from corresponding ones in Fig. 2. Curve A shows small heat absorption at about 65°C. Along curve B the heat absorption occurs twice at

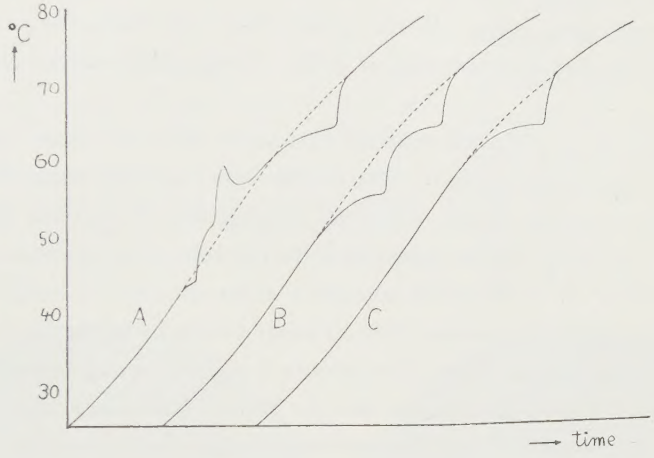

Fig. 3. Heating curves
(A) For the rapidly cooled sample,
(B) For the slowly cooled sample,
(C) For the sample cooled very slowly from the melt, or kept at the elevated temperatures after solidification.

about 56.5°C and 65°C. If the drop has been cooled too slowly, the sample becomes opaque. For such sample, the heat absorption was observed only once (curve C).

According to the results of microscopic and electron diffraction studies the heat absorptions at about 44°C, 56.5°C and 65°C are due to the melting of three modifications and we shall call them respectively α form, β' form and β form after Lutton (**5**). Figs. 2 and 3 show that the three forms can be realized by rapid, slow and extremely slow cooling respectively. The heat evolution peak on the heating curve A may be explained as follows : the first step corresponds to the heat evolution due to the crystallization of β' phase in the melt of α and this increase in temperature of the sample induces further transformation of β' into the most stable β phase. The heat of transition is responsible for the second step of the peak. The temperature at the top of the first step is considerably lower than the β' melting point (56.5°C) and varies according to the mass of the sample, so the separation of the steps is not necessarily caused by the melting of β' phase.

The modifications appeared in the heating processes were identified by electron diffraction method. To observe the change in the transmission pattern, the specimen spread on the gold-leaf was heated in the diffraction apparatus, and three typical processes were observed according to the previous heat treatment. Starting with α

form, as increasing temperature, the liquid pattern instantaneously appears at about 44°C, and then β' and β patterns appear successively, the last one remaining until 65°C. The pattern observed at the time of β'–β transition differed from either of β' and β, but it is uncertain whether this intermediate state is another modification or only a mixture of these forms. If the initial state is β' form, the pattern remains unchanged until the liquid is observed at 56°C. The β pattern never changes below 65°C.

Next, the change in dielectric constant associated with the phase transformation was studied by the tuning method. The results were consistent with those from the thermal analysis, although details could not be detected on account of large heat capacity of the condenser plates compared with the substance between them. Fig. 4 shows the behaviour of the dielectric constant ε when the melt is rapidly cooled and then heated. The value of ε increases with increasing temperature below the α-melting point. This may be related to the orientational freedom of polar segments which has been suggested by Crowe and Smyth (**2**). In Fig. 5, when the melt is cooled down to about 50°C and is kept at this temperature, ε begins to decrease and tends to the value for β phase. If the specimen is heated again before ε could reach the final value, a peak is observed at β'-melting point, suggesting the melting of β' followed by the crystallization of β form. The temperature dependency of ε of β' form could not be observed, because this phase is so changeable that it is scarcely isolated from other phases. Microscopic observation gave some results about the stability of β' phase. Fig. 6 shows the formation of β phase by very slow cooling at the temperature above the melting point of α, and the poor temperature dependency of ε was observed for this phase suggesting the lack of the dipole rotation because of the complicated crystal structure. These three curves, i.e. Figs. 4, 5, 6,

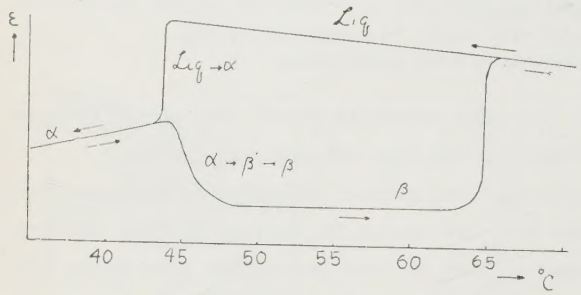

Fig. 4. ε–temperature diagram for rapid cooling and subsequent heating.

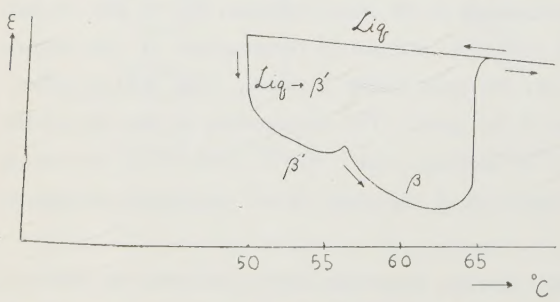

Fig. 5. Cooling was stopped at 50°C and the temperature was kept constant for 30 minutes and then heated. The curve shows some increase in ε due to β' melting.

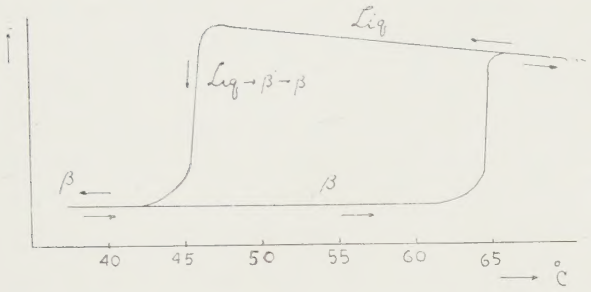

Fig. 6. On slow cooling, ε began to decrease remarkably at 46°C due to the growth of β′ form, but it then transformed to β form at this temperature and on heating ε did not show any sign of the existence of the metastable phases.

correspond to the curves A, B, C in Fig. 3 respectively.

Plate 1 represents the cross nicol pattern of α phase formed between two glass plates. If the solidification occurs at a temperature just below α melting point, α form makes spherulites, but only irregular aggregates are produced by too rapid cooling. When the temperature of

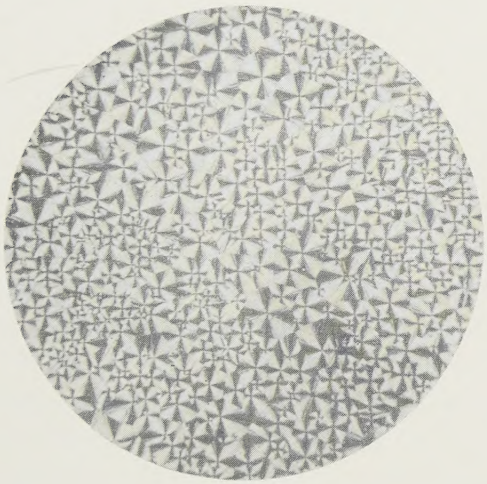

Plate 1.

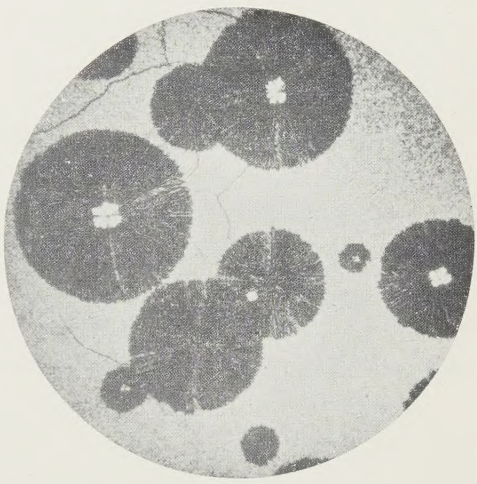

Plate 2.

undercooled liquid is kept between two melting points of α and β′ forms, spherulites of β′ form grow. Plate 2 represents β′ phase formed in a thin layer at 52°C. The back-ground is α phase formed by subsequent chilling. In this temperature range both the rate of growth and the rate of nucleation of β′ phase increase with decreasing temperature. In Fig. 7, curve A is a plot of the rates of increase in diameter of the spherulite as a function of temperature. The slope of the curve varies remarkably at about 46°C. But it was observed that the growth of the spherulite is caused by the growth of individual crystals at higher temperatures and by the formation of new crystals of its surface at low temperatures. Accordingly the high-temperature part of this curve represents the growth rate of the single crystal in the most rapid direction. On the other hand, the large rate at lower temperature may be attributed to some factors. These two types of β′ form differ not only in size of the crystals but also in

melting point, the high-temperature type being more stable. Nevertheless no striking difference was found between them in the X-ray diffraction pattern except in diffuseness of the lines, and so both were treated as β' form. During the growth of β' phase, β' to β transformation proceeds, too. In Plate 2, the white spots near the centre of each circle represent those parts which have already transformed to β form. The growth rate of β crystal in this transformation is also shown by curve B in Fig. 7. Above the β' melting point, β phase grows only when it already exists in the melt, but spontaneous nucleation often occurs in a quite different way. When the melt is undercooled below the β' melting point, a cloud of minute crystals originates and spreads out at a rate far greater than the growth rate of individual crystals. The crystallization is completed in a short time and β phase is produced. In this case, the rapidity of the crystallization seems to be caused by very frequent nucleation stimulated by the motion of the minute crystals.

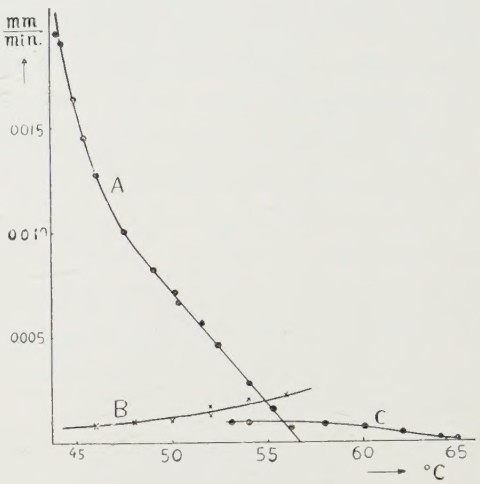

Fig. 7. The growth rate of the crystal
(A) β' sphurlite in the liquid phase,
(B) β crystal in β' phase,
(C) β crystal in the liquid phase.

At last, each modification was studied by X-ray diffraction. The spacings obtained from the powder photographs are given in Table 1. The diffraction lines due to α form were more or less diffuse, while the pattern due to β form was very complicated and the values of the spacing are not exact. For example, the strongest line could be separated into two lines of nearly equal intensity by rotating crystal method. Disregarding such details, the three modifications are characterized respectively by one, two and three strong diffraction lines. Clearly they correspond to the possible three modes of side packing of the long chains of the molecule, namely, hexagonal type, orthorhombic or monoclinic type and triclinic type respectively. Of course these types are common to aliphatic long chain compounds. If grycerol groups and other details are disregarded, it is possible to construct the crude models of the three forms after the examples of single chain compounds. In Fig. 8 the unit cells of such models are represented in cross-sections. The cross-section per hydrocarbon chain amounts to about 19.8Å^2, 19.2Å^2 and 19.1Å^2 for the three forms respectively. Considering that the molecule of triglyceride consists of three chains, the true unit cells of β and β'

Table 1.

α form (vertical)		β′ form (tilted)		β form (tilted)	
45.2	v. s.	42.0	v. s.	40.3	v. s.
15.2	s.	13.8	s.	13.45	s.
9.06	w.	8.5	w.		
7.59	w.	7.05	w.	6.70	w.
5.70	w.	5.37	w.	5.03	w.
4.15	v. s.	4.21	v. s.	4.56	v. s.
2.39	w.	3.78	s.	3.84	s.
2.10	v. w.	3.07	m.	3.68	s.
		2.79	w.	3.41	m.
		2.54	m.	2.56	s.
		2.25	m.	2.35	m.
		2.07	w.	2.26	m.
		1.88	w.	2.18	m.
		1.77	m.	2.12	m.
				2.06	m.
				1.95	m.

forms must have areas at least 3/2 times as large as the corresponding units in Fig. 8. Vand and Bell (**4**) obtained more detailed figure on the crystal structure of β form of trilaurine. In α phase, however, it is possible to construct one molecule with any three neighbouring chains on account of the high symmetry of the crystal. So any conclusion on the regularity of the arrangement of the glycerol groups in α phase, is difficult to deduce.

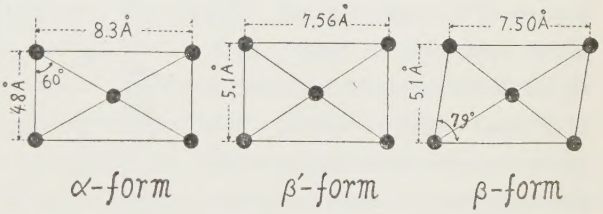

Fig. 8. Cross-sections of the units of three forms which correspond to the unit cells of paraffins.

3. Discussion

The symmetry of α form may be described as being hexagonal. In general, such symmetry of vertical forms is associated with the rotational freedom of the linear chain molecules around their long axes. But triglyceride molecules are not linear. Clarkson and Malkin (**1**) concluded from their X-ray investigation for long spacings the shape of the triglyceride molecule to be like a tuning fork. The three chains in a molecule are joined to the glycerol group with C–O–C bonds. If the bonds were sufficiently flexible, the individual chains would be allowed to rotate in α phase, just

as the linear molecules in the high-temperature modifications of hydrocarbons. But the triglyceride molecule is composed of three chains connected to the glycerol group, so the rotation of the individual chain does not necessarily bring the hexagonal symmetry of the crystal structure as in the case of the linear chain molecule. It is rather reasonable to think that the hexagonal symmetry of α phase is due to the disordered arrangement of the molecules inherited from the liquid phase, so in this case the rotational motion of individual chain is not the necessary condition for the hexagonal symmetry. The fact that no marked phase transformation to non-rotating ordered state occurs by further cooling, which is usually observed for the molecular rotating phase of the linear chain molecule, seems to support this view point.

For β' form, which is monoclinic or at least nearly monoclinic, the chains are considered to be most commonly arranged. Such configuration of the chains will require the regular arrangement of molecules. But since the cross-section of the chain (about 19.2Å^2) is somewhat larger than usual (about 18.5Å^2), some degree of orientational freedom of the chains around their axes may occur.

β form is clearly triclinic and is most peculiar modification, though a few examples of other long chain compounds have similar structures.

Fig. 9 represents the schematic process of the triple melting. The liquid phase can be easily undercooled until the solid modification of highest entropy α grows. This phase remains unchanged provided the temperature is sufficiently low. On heating such sample settled in α phase, it once melts at its melting point (A), leaving seeds of β' form as seen from the microscopic observation. They soon grow up into β' phase. On futher heating, it changes again to β phase in the same manner at the β' melting point (B). The last phase melts at the highest melting point (C). Actually this process must be disturbed by the transforamation between solid phases.

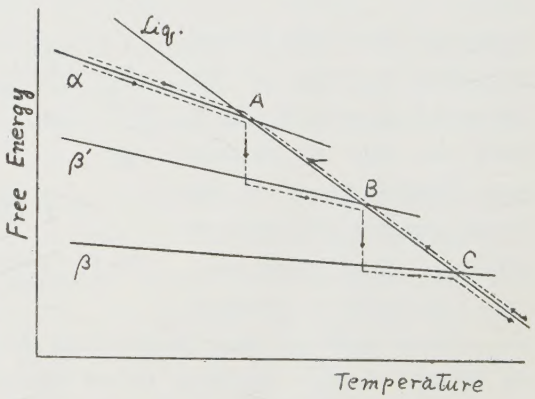

Fig. 9. Free energy *versus* temperature diagram for the liquid and three solid phases. Dotted line shows the triple melting process.

For most aliphatic long chain compounds the relative stability of polymorphic forms at the melting point is delicate, while that of triglyceride shows the distinct tendency that the form with higher symmetry is less stable. So far as the arrangement of the chains is discussed, the crystal structure of each modification of tripalmitin compared

with other long chain compounds has not particularity to be specially mentioned. The long chain parts of triglyceride are forced to be packed in a manner similar to that for single chain molecules, the presence of glycerol links combining each one of three chains resists to this tendency and will cause the deformation of the molecules from their stable configuration and will limit the freedom of the chains, that is, the linkage will tend to raise the free energy of the modification relative to that which would be expected for linear chains. This effect of the linkage on the stability may be large for the high-symmetry form, the structure of which is favourable to free chains, but the effect must be minimized for low-symmetry form where the molecules are fixed in the most stable positions. Consequently the free energy of high-symmetry phase may be raised relative to that of the low-symmetry phase. If this effect is comparable to that for liquid phase, the high-symmetry phase will lose its stable region.

From the standpoint of phase transformation, the shape of the molecule is evidently favourable to disordered phase, because the probability that molecules are placed in order is very small even if the chain parts of them are almost regularly arranged. If α form permits irregularity in the orientation of the molecules, the growth of this phase may be accomplished without interchange in the position of neighbouring chains, since the chains are expected to be nearly parallel to each other in supercooled liquid. While the molecules are regularly arranged in β phase and perhaps in β' phase, so the rates of nucleation and growth of these phases must depend on the symmetry and the shape of the molecule. At lower temperatures, the mobility of the molecules become important, because the rotation of the molecule which is necessary for the growth of ordered phase will require the interchange of the position of neighbouring chains. This means that the growth of the disordered phase becomes dominant with decreasing temperature.

The phase transformation between solid phases will be accompanied by another difficulty which arises from the difference in tilt angle. In fact, the slowness of such transformation has been observed even in single chain substances such as higher alcohols. In triglyceride, in addition to the above-mentioned difficulty the glycerol rinks may be responsible for the occurrence of the metastable phases.

Besides the phenomenon of multiple melting, tripalmitin has another peculiarity in polymorphism. Solid samples exhibit different properties depending on the difference in the temperature at which they have grown, even if they assume the same polymorphic form. β phase which has transformed from β' phase, melts at a temperature lower than the true melting point observed on that which has grown slowly from the melt. Lutton (**5**) found that the melting point of β phase just formed can be raised by keeping its temperature near the melting point. The melting point and the sharpness of the X-ray diffraction lines of β' form increase with the temperature at which

this phase has grown. Probably the distinction between high- and low-temperature types of this form is not so essential. The melting point of α phase could not be measured accurately on account of the rapid transformation to more stable forms near the melting point, but the breadth of diffraction lines shows the same tendency as β' form.

These facts suggest the presence of the lattice imperfections. When the crystal grows at lower temperatures, the faults at which some molecules are fixed in wrong orientation will occur more frequently than at higher temperatures. Such imperfection will cause the decrease in melting point, since the imperfection may be expected to last until the temperature approaches the melting point.

Acknowledgement

In conclusion, the authors wish to express their thanks to Prof. K. Tanaka and Prof. R. Goto for their helpful discussions and advices. This work has been carried out by a grant-in-aid of the Ministry of Education.

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STUDY ON CHANGE OF GRAVITY WITH TIME
PART III. ON THE TIDAL FACTOR OF GRAVITY

BY

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(*Received April 26, 1956*)

ABSTRACT

For the purpose of determining the accurate value of the tidal factor of gravity ($G \equiv 1-3k/2+h$; h, k: Love numbers), the one month's gravimetric observations were made at three stations of Kyoto, Chikubushima, and Shionomisaki where the effects of the ocean tide and geological structure, if existing, upon the tidal factor value were considered to be representatively different from each other. As deduced from the analysis of the present observations and after the calculation of disturbance effect, the most valid value of G was estimated as 1.18 ± 0.02. And by combining the present value of $G=1.18$ and $D=0.66$ ($D=1+k-h$) derived from the tiltmetric observation on earth tide, the Love number values were calculated as $h=0.66$, and $k=0.32$, which gave the earth tidal upheaval of the ground of more than 50 cm in the maximum. Moreover, in the present article the gravity variation connected with the variation of atmospheric pressure is discussed in some detail.

1. Introduction

(1) *Outline of the theory of earth tides*

Detailed theoretical investigations on the phenomena of earth tides have been fully made by Lord Kelvin, A. E. H. Love, H. Takeuchi, and many others, as briefly described in Part I of this series of research. Though it is not the purpose of the present article to describe the details of those theoretical treatments, it is considered to be convenient to describe an outline of the theory of earth tides to clarify the aim of the present investigation and the significance of the results obtained in our observation.

Let the tide-producing potential at any point within or near the earth due to a celestial body be denoted by W_2, then W_2 can be expressed with sufficient approximation by the formula

$$W_2 = \frac{1}{2}\gamma M D^{-3} r^2 (3\cos^2\theta - 1)\,, \qquad (1)$$

where γ denotes the constant of gravitation, M the mass of the celestial body, D the distance between the centres of the earth and the celestial body, r the distance of the point from the earth's centre, θ the angle between the radius vectors drawn from the

earth's centre to the point and to the centre of the celestial body. Then W_2/g expresses the theoretical equilibrium height of the ocean tide caused by the celestial body, g being the mean value of gravity at the earth's surface. This theoretical equilibrium height of the ocean tide can be calculated under the assumption that the earth is perfectly rigid and therefore it does not yield to the tidal force. Assuming that the distribution of mass in the earth's interior is spherically symmetrical, and that the rigidity and incompressibility are constant over the same surface as the density (if the rotation were taken into account, the spherical surfaces of equal density would have to be replaced by ellipsoidal surfaces), the radial displacement U of any point of the solid earth, which is produced by the force derived from the potential W_2, ought to be expressed by the product of W_2 and a certain function of r; thus we may write

$$U=H(r)\frac{W_2}{g}, \tag{2}$$

where the function $H(r)$ depends upon the distribution of density and elasticity in the earth. The potential due to the increment, or decrement, of density that accompanies the cubical dilatation, and to the superficial displacement of matter, ought to be also expressed by the product of W_2 and some function of r, so that we may write for the potential of the earth deformed by the tidal force an expression of the form

$$V=V_0+K(r)W_2, \tag{3}$$

where V_0 denotes the potential of the undisturbed earth. Now denote the mean radius of the earth by a and write

$$h=H(a), \qquad k=K(a),$$

then h and k are two numbers which define the height of the earth tide at the earth's surface and the additional potential that is produced by the earth's deformation, and are usually called "Love numbers". Using these Love numbers, for the surface of the earth we have

$$U=h\frac{W_2}{g}, \tag{4}$$

$$V=V_0+kW_2. \tag{5}$$

Combining (5) with W_2 in (1), we get the total potential due to the deformed earth and the original tide-producing potential, which is expressed by

$$V_0+kW_2+W_2.$$

From this we get for the equilibrium height of the ocean tide, referred to the undisturbed earth, a formula of

$$(1-k)\frac{W_2}{g}.$$

Accordingly, if the height of the ocean tide is observed at a point of the deformed earth's surface, which is expressed by $r=a+h\frac{W_2}{g}$, its height ought to become

$$(1+k-h)\frac{W_2}{g};$$

in other words, the equilibrium height of the ocean tide is modified in the ratio $(1+k-h)/1$ to that for a perfectly rigid earth, in consequence of the existence of the earth tide. The number $D\equiv 1+k-h$ is usually called "diminishing factor".

The diminishing factor D is also obtained by observation of the deflection of plumb-line. The amount of deflection of plumb-line, or change of the direction of gravity, caused by tidal force ought to be diminished in the same ratio D in consequence of the deformation of the earth. It will be understood by the following. As was already seen, the surface of the deformed earth is expressed by the equation

$$\varphi(r,\theta)\equiv r-a-h\frac{W_2}{g}=0; \tag{6}$$

on the other hand, the level surface defined by the total potential due to the deformed earth and the original tide-producing potential is expressed by the equation

$$\psi(r,\theta)\equiv V_0+kW_2+W_2+\text{const.}=0. \tag{7}$$

The angle between the normal lines of these two surfaces is that which is observed with the tiltmeters settled on the earth's surface as the tidal deflection of the plumb-line. The angle α_1 between the radius vector and normal line of the surface (6) at a point is calculated by the expression

$$\alpha_1=-\frac{1}{r}\frac{\dfrac{\partial\varphi}{\partial\theta}}{\dfrac{\partial\varphi}{\partial r}}=\frac{h}{ag}\frac{\partial W_2}{\partial\theta}; \tag{8}$$

similarly, the angle α_2 between the radius vector and normal line of the surface (7) at the same point is calculated by the expression

$$\alpha_2=-\frac{1}{r}\frac{\dfrac{\partial\psi}{\partial\theta}}{\dfrac{\partial\psi}{\partial r}}=\frac{1+k}{ag}\frac{\partial W_2}{\partial\theta}. \tag{9}$$

Subtracting α_1 from α_2, its remainder becomes

$$(1-k-h)\frac{1}{ag}\frac{\partial W_2}{\partial\theta},$$

which is the tidal deflection of plumb-line to be observed actually. If the earth were perfectly rigid, it would be $\frac{1}{ag}\frac{\partial W_2}{\partial \vartheta}$. Accordingly the ratio of the observed deflection of plumb-line to that which would occur if the earth were perfectly rigid is also $(1+k-h)/1$.

Now, the tidal variation of the intensity of gravity in case that the earth yields to the tidal force can be derived by the following processes. The potential at a point on the surface of the deformed earth is considered to consist of the following four parts:

(i) potential of the undisturbed earth V_0,

(ii) potential induced by the displacement of the point $U\frac{dV_0}{dr}$,

(iii) potential induced by the deformation of the earth $K(r)W_2$, and

(iv) original tide-producing potential W_2.

Formularizing this, it becomes

$$V=V_0+U\frac{dV_0}{dr}+K(r)W_2+W_2. \tag{10}$$

Differentiating (10) with respect to r, we get for the variational part of gravity Δg an expression of the form

$$\Delta g=U\frac{d^2V_0}{dr^2}+\frac{\partial}{\partial r}(K(r)W_2)+\frac{\partial W_2}{\partial r}. \tag{11}$$

From Laplace's equation

$$\nabla^2 V_0=\frac{d^2V_0}{dr^2}+\frac{2}{r}\frac{dV_0}{dr}=0,$$

we get

$$\frac{d^2V_0}{dr^2}=-\frac{2}{r}\frac{dV_0}{dr}. \tag{12}$$

Next, since $K(r)W_2$ is proved to be approximately proportional to $\frac{1}{r^3}$, we get

$$\frac{\partial}{\partial r}(K(r)W_2)=-\frac{3}{r}(K(r)W_2). \tag{13}$$

Lastly, by differentiating the expression (1) we get

$$\frac{\partial W_2}{\partial r}=\frac{2}{r}W_2. \tag{14}$$

Inserting (12), (13), and (14) into (11), and taking into account that $H(r)=h$, $K(r)=k$ for $r=a$, and $\left(\frac{dV_0}{dr}\right)_{r=a}=-g$, we get

$$\Delta g=(2h-3k+2)\frac{W_2}{r}. \tag{15}$$

Inserting (14) into (15), we obtain the final result for the earth's surface

$$\Delta g=\left(1-\frac{3}{2}k+h\right)\left(\frac{\partial W_2}{\partial r}\right)_{r=a}.$$

If the earth were perfectly rigid, h and k would be zero and Δg become $\left(\frac{\partial W_2}{\partial r}\right)_{r=a}$. Accordingly the tidal variation of gravity intensity on the surface of deformed earth ought to be reduced in the ratio $\left(1-\frac{3}{2}k+h\right)/1$. The number $G\equiv 1-\frac{3}{2}k+h$ is usually called "tidal factor of gravity" or, in short, "gravimetric factor".

If the diminishing factor D is known from observations of the ocean tide or tiltmetric observations and the gravimetric factor G is known from gravimetric observations, we can easily determine, without any assumption on the distribution of density and elasticity of the earth's interior, the values of Love numbers h and k individually by algebraic calculation. But, as the next step, it is theoretically very difficult to calculate the distribution of density and elasticity of the earth's interior from the obtained values of Love numbers. In case of a simplified earth model under the assumption that the matter within the earth is absolutely incompressible and the density is uniform, we should have

$$k=\frac{3}{5}h; \tag{16}$$

if, further, the rigidity is assumed to be also uniform, we have the following relation

$$h=\frac{5}{2}\left(1+\frac{19}{2}\frac{\mu}{\rho g a}\right)^{-1}, \tag{17}$$

where ρ and μ denote the mean density and mean rigidity of the earth respectively. For a complex model of almost real earth, it needs an elaborate numerical calculation to determine the distribution of density and elasticity with the aid of knowledges obtained from the geodetic and seismological observations, and moreover the result thus obtained contains many undesirable uncertainties.

(2) *Purpose of the present investigation*

In spite of such difficulties as mentioned above, the diminishing factor D and gravimetric factor G undoubtedly supply us very important information on the elasticity of the matter within the earth. On account of this importance, a large number of investigators have made efforts to determine the most reliable values of D and G since the time of Lord Kelvin. In spite of their extraordinary efforts, various difficulties annexed to the observations have, especially in case of the gravimetric factor, disturbed the accurate determination of the values of D and G. The difficulties in the determination of the diminishing factor exist mainly in the fact that the actual height of the ocean tide is far apart from that which is expected from the equilibrium theory of the ocean tide, and in the fact that in case of tiltmetric observations the existence of the ocean tide produces baneful effects upon the study of the pure earth

tide. From this point of view, the tiltmetric observations by A. A. Michelson (**1**) and E. Nishimura (**2**) which were made at the observation stations far apart from the ocean are considered to be ones to give the most reliable value of D. On the other hand, the determination of the gravimetric factor had been left behind that of the diminishing factor, chiefly because of instrumental difficulties with the gravimeter available for the purpose. The appearance of portable gravimeters which were rapidly developed since the end of the Second World War made it covenient not only for exploration but for the study of the earth tide; however, all the values of G obtained in the past days with portable gravimeters and station gravimeters are, as were shown in Table 1 of Part I of this series (**3**), greatly diverse from each other. The causes of diversity of those G–values are considered to exist, as mentioned in Part I, partly in the secondary effects of the ocean tide and the effects of local geological structure; however, it should be emphasized that the difference of the length of observation period also produces considerable effects on the G–value. As it will be illustrated by numerical results in the following section, there is great difference between the G–values obtained by observations for each period of a month, a fortnight, and a week, even in regard to M_2–component; and it is generally concluded that it needs the observation of at least a month or more in order to get a reliable G–value of the M_2–component.

Considering the above-mentioned circumstances on observation period and the effects of the ocean tide and local geological structure, it is concluded that, in order to examine these effects and obtain an accurate value of G, it must be observed for period of one month at least at many observation stations under various conditions. Under these circumstances the observations of the tidal variation of gravity were made at three stations under different conditions in the Kinki District for period of one month respectively. In the present article, the values of G obtained from each observation and the most reliable value of G obtained after examination of the above-mentioned effects will be reported, and further the values of Love numbers h and k will be computed.

2. Observations

(1) *Observation stations*

In order to accomplish the above-mentioned purposes, observation stations where the effects of the ocean tide and geological structure are possibly large ought to be selected. In regard to the meaning of "geological structure", it must be noticed that the "geological structure" which should affect the value of G, means such one of large scale as the thickness of the earth's crust or the tectonic differences between the continent and the ocean. From this point of view, it is considered to be most effective

for the present purpose to observe at any number of stations on the continent as well as on solitary islands in the far-off ocean. But in the present state of our country, the accomplishment of this sort of observation is extremely troublesome and practically

Table 1.

	Station	Latitude (N)	Longitude (E)	Height (m)	g (gal)	BOUGUER anomaly (mgal)
I	Kyoto	35°02′	135°47′	58	979.722	− 12
II	Chikubushima	35°25′	136°09′	98	979.714	− 45
III	Shionomisaki	33°27′	135°46′	74	979.740	+139

I	Observation period	June 11th 12h 00m～July 13th 05h 00m, 1954
II		Sep. 28th 15h 00m～Oct. 30th 12h 30m, 1955
III		Nov. 16th 09h 00m～Dec. 17th 12h 00m, 1955

I	Kyoto University, Geophysical Institute, No. 22 Laboratory, on the concrete block for experiment.
II	Hōgonji Temple, The Office of Temple, on the earth floor of the side entrance.
III	Shionomisaki Weather-station, Seismograph Room, on the surface of the concrete block.

Fig. 1. Positions of the observation stations, and distribution of BOUGUER anomaly in mgal.

impossible; therefore, as a second best plan, some stations where the thickness of the crust is considered to be different from each other in our country were selected for the present gravimetric observation. Concerning the problem of the thickness of the crust in our country, the "map of BOUGUER anomalies" compiled by C. Tsuboi (**4**) gives the most precise information. Considering the distance from the Pacific Ocean shore in regard to the effects of the ocean tide, BOUGUER anomalies in regard to the effects of geological structure, and some other circumstances, three stations of Kyoto, Chikubushima in Lake Biwa, and Shionomisaki of the Kii Peninsula were finally selected as the most suitable ones for the present purpose. The locations of these stations, the observation periods and others are as shown in Table 1. Fig. 1 shows the relative positions of these stations for the aspect of BOUGUER anomalies. The variations of temperature and atmospheric pressure in each observation room are shown in Fig. 2. Hereafter, these three stations and the observations made at these stations will be called the station I, II, III, and the observation I, II, III respectively in this sequence.

(2) *Instrument*

All of the present observations were made by the use of the same instrument, a Worden Gravimeter No. 127. Its sensitivity given by the factory is 0.18473 mgal/div., and for practical purposes it can be read to one tenth of the scale division by the vernier. According to the experiments on the reliability of the scale constant made by the comparison with North American Gravimeter AG-1, 108, the accuracies of the scale constants of the both gravimeters accord well with each other within the limits of experimental errors. Although the problem concerning the absolute value of the gravimeter scale constant is generally very troublesome owing to the difficulties in the method of check, it may safely be said from various considerations that the scale constant given by the factory is trustworthy at least to the third significant figure. Consequently, in case of discussing the tidal factor of gravity to its third significant figure, the value of scale constant given by the factory will correctly be adopted in the present observation. On the other hand, from our other experiments and the experiments made independently by the members of the Geological Institute of Kyoto University, the above-described scale constant has remained constant within the limits of experimental errors since the time of its importation (1953); in other words, there is no noticeable time variation in the scale constant of that gravimeter. The plastic flow of the spring, or simply expressed as drift, of the gravimeter was fairly large, and moreover, the drift speed was never uniform. The subject of non-uniformity of the drift speed will be discussed in some detail in the next section, together with the influences of the meteorological factors upon the gravimetric observation. The

average values of drift speed were estimated, in the present case, to be 0.600 mgal/day, 0.935 mgal/day, and 0.863 mgal/day for the periods of the observations I, II, and III respectively.

3. Results of the observations

The gravity variations at each station were read at intervals of half an hour, those values being collectively shown in the table at the end of the present article. Although the readings were made at intervals of half an hour, the hourly values alone were used for harmonic analysis under considerations of all the values. At the first step to enter harmonic analysis, it needs the reduction of drift. Since the drift speed was, as already mentioned, not uniform even in each observation period, the reduction was done by subtracting the running means of the observed values of 25 hours from the corresponding observed values in the middle of the 25 hours period. The curve joining the points which correspond to the values of running means represents the aspect of the drift, and the drift curve thus obtained was not a linear and monotonously increasing line in the present case. Since the proper plastic flow of the spring is perhaps linear with time for the period of only one month, the deviation of the actual drift curve from a straight line is considered to be affected by some external causes other than the proper plastic flow of the spring, for example such as meteorological factors. And so, before beginning the harmonic analysis, the correlation between the drift and meteorological factors should be examined.

(1) *Correlation between the drift and meteorological factors*

The uppermost curves at each station in Fig. 2 represent the deviations of actual drift curves from the straight lines corresponding to the mean drift speeds described in the preceding section; and the middle and lowermost curves represent the variations of the atmospheric pressure and temperature respectively. Since the variations of atmospheric pressure and temperature at station I were observed by reading the scales of a siphon mercurial barometer and a thermometer, and those at stations II and III were read from the records of a self-recording barograph and thermograph, some differences are to be noticed between the modes of the variations of those at stations I and II, III; but the differences are small and negligible for the purpose of discussing the general aspects of variations. Comparing the three curves of drift, pressure, and temperature with each other, it will be recognized that there exists generally a fairly good correspondence between the curves of drift deviation and atmospheric pressure for every station, though it is not always so in the details of variation. Furthermore, the both variations show the same sense. This fact contains an important significance in itself, for it represents that the increase or decrease of atmospheric pressure

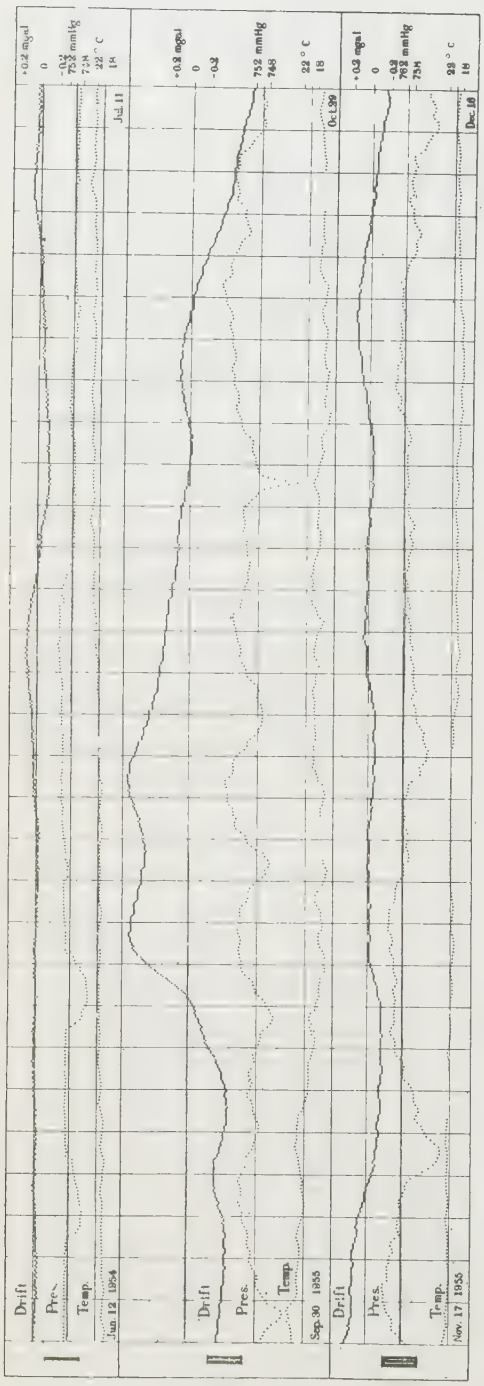

Fig. 2. Drift residuals in the observations at the three stations, and changes of atmospheric pressure and temperature.

apparently cause the increase or decrease of gravity. The phenomenon is entirely similar to that which was observed in the case of gravimetric investigation with the double bifilar gravimeter designed by the present writer (**3**). As described in that occasion, if it is assumed that the increase or decrease of atmospheric pressure corresponds to the increase or decrease of density of air, by whatever manner of the effect caused by the variation of buoyancy for the bob of gravimeter or the upward attraction of air mass, the increase or decrease of atmospheric pressure should produce the decrease or increase of gravity; but in practical case the effect is entirely reversed. From the fact that such curious phenomena as abovementioned were observed with two kinds of gravimeters having structures entirely different from each other, it is possibly concluded that the residual drift or, if a simple expression be permitted, the nonperiodic variation of gravity thus observed is caused by some unkown mechanism other than the effects of buoyancy and attraction of air mass. Upheaval or subsidence of the earth's surface caused by the variation of atmospheric pressure, which accompanies the vertical displacement of observation station as a necessary conse-

quence, is considered to be a possible mechanism explaining the phenomena. If the phenomena really originate from such a mechanism, it will afford an important clue for study of the elastic properties of the earth's crust; on the other hand, however, if it is assumed that the phenomena originate merely from such a mechanism, the fact shown in Fig. 2 requires the earth's surface to ascend or descend to the extent of 1~3 meters for the atmospheric pressure variation of 1 cm Hg; and such an enormous change is, at present, beyond our common knowledge. The relation of the variation of atmospheric pressure to the movement of the earth's surface is a very attractive problem, but its complete solution should await the further development of the gravimetric observation, and consequently the consideration of the nature of residual drift discussed above will be postponed to future investigation.

Now, although it is the matter little connected with the theme of the present article, some descriptions need to be made concerning the gravity variation at the

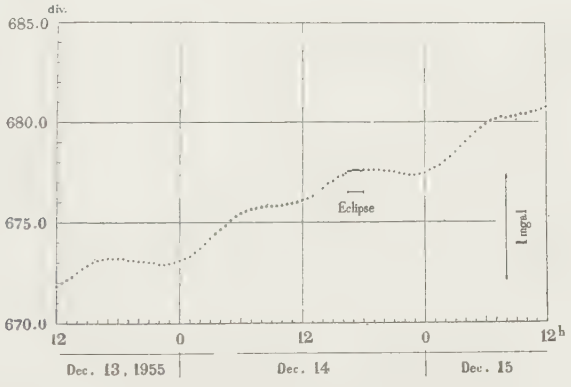

Fig. 3. Change of gravity at Shionomisaki before and after the time of the solar eclipse.

time of solar eclipse. Observation of the gravity variation at the time of solar eclipse contains an important and attractive significance, because it is related to the problem whether the gravitational attraction of the sun is screened or not, however slightly, by the moon. From the data of his observation on gravity variation during the solar eclipse on June 30, 1954, R. Tomaschek (**5**) has reported the negative result in regard to the screen effect of solar gravitational attraction by the moon. In course of the present observation III, at Shionomisaki, the partial solar eclipse was observed on December 14, 1955. The eclipse, being on the wane of nearly a quarter, continued from about 16h 32m till sunset; the gravity variations during the eclipse were especially read at short intervals of 15 minutes. As shown in Fig. 3, some slight influence of the solar eclipse upon the variation of gravity may be traced by partial eyes of

looking, but this attractive problem on the "screen" effect of gravitation and other particulars will be fully discussed after the accomplishment of observation with an evacuated gravimeter of extremely high sensitivity of 10^{-10} mgal/mm which is now in course of construction.

(2) *Tidal factor of gravity*

After the application of reduction for drift, the gravity values observed were harmonically analyzed to obtain the values of component tides. The observation periods used for analysis are as follows:

at the station I, June 11th 15h~July 11th 15h, 1954;
at the atation II, Sep. 29th 15h~Oct. 29th 15h, 1955;
at the station III, Nov. 16th 15h~Dec. 16th 15h, 1955,

in Greenwich Mean Time respectively. Harmonic analysis was made for the component tides of lunar semi-diurnal M_2, solar semi-diurnal S_2, lunar diurnal O_1, and lunisolar diurnal K_1. It is to be remarked that values for S_2, O_1, and K_1 are considered to be certainly less accurate compared with that for M_2 because of the short analysis period of one month among other reasons. Expressing the results of harmonic analyses for each component tide by the form of $R_{obs}\cos(nt-\zeta_{obs})\pm\delta$, R_{obs}, ζ_{obs}, and δ are as shown in Table 2, where ζ_{obs} denotes the time interval from the instant when the imaginary celestial body corresponding to that component tide had transitted the meridian of that observation station till the instant when the observed gravity actually reached the maximum value, and δ signifies the mean error which mainly originated in the accuracies of the gravimeter and reading techniques. The

Table 2.

	M_2			S_2			O_1			K_1		
	R_{obs} (μgal)	δ (μgal)	ζ_{obs}	R_{obs} (μgal)	δ (μgal)	ζ_{obs}	R_{obs} (μgal)	δ (μgal)	ζ_{obs}	R_{obs} (μgal)	δ (μgal)	ζ_{obs}
I	58.9	0.6	181°.96	27.2	0.5	176°.67	36.1	1.1	178°.07	55.6	2.7	202°.73
II	60.6	0.5	181°.20	28.5	0.4	182°.22	32.5	1.3	174°.30	53.6	5.3	180°.95
III	64.6	0.7	182°.42	30.0	0.4	185°.37	27.7	1.2	179°.61	54.4	3.8	199°.62

values in Table 2 show the actual variations of gravity in those observation periods; but it is convenient for succeeding discussions to reduce these values of lunar components to ones in the case that the inclination of the moon's orbit to the equator is equal to its average value of 23°27′. Expressing such reduced results by the form of $H_{obs}\cos(nt-\zeta_{obs})\pm\varepsilon$, H_{obs}, ζ_{obs}, and ε are as shown in Table 3. As natural results of the reduction, those values of H_{obs} do no longer include in themselves the factors originating from the difference of the epochs of observations; therefore, the difference between those values of H_{obs} in the same component tide is

entirely attributed to the local effects comprising the difference of the latitudes of the observation stations.

Table 3.

	M_2			S_2			O_1			K_1		
	H_{obs} (μgal)	ε (μgal)	ζ_{obs}	H_{obs} (μgal)	ε (μgal)	ζ_{obs}	H_{obs} (μgal)	ε (μgal)	ζ_{obs}	H_{obs} (μgal)	ε (μgal)	ζ_{obs}
I	59.4	0.6	181°.96	27.2	0.5	176°.67	33.7	1.1	178°.07	53.3	2.7	202°.73
II	60.2	0.5	181°.20	28.5	0.4	182°.22	32.8	1.3	174°.30	54.1	5.3	180°.95
III	64.0	0.7	182°.42	30.0	0.4	185°.37	28.2	1.2	179°.61	54.1	3.8	199°.62

On the other hand, the theoretical values of tidal variation of gravity—the values of tidal variation of gravity which would occur if the earth were perfectly rigid or it does not entirely yield to tidal force—can be theoretically computed from the results of astronomical and geodetic observations. As the most appropriate numerical values of the constants necessary for the computation the values quoted in the paper by J. T. Pettit (**6**) were adopted. The theoretical values of H_{theor} and ζ_{theor} corresponding to H_{obs} and ζ_{obs} are as shown in Table 4. Difference beteen the values

Table 4.

	M_2		S_2		O_1		K_1	
	H_{theor} (μgal)	ζ_{theor}	H_{theor} (μgal)	ζ_{theor}	H_{theor} (μgal)	ζ_{theor}	H_{theor} (μgal)	ζ_{theor}
I	50.3	180°.00	23.4	180°.00	29.3	180°.00	41.2	180°.00
II	49.8	180°.00	23.2	180°.00	29.4	180°.00	41.4	180°.00
III	52.2	180°.00	24.3	180°.00	28.6	180°.00	40.3	180°.00

of H_{theor} of the same component tide in Table 4 is entirely attributed to the difference of the latitudes of the observation stations. The tidal factor of gravity G is obtained by taking the ratio of H_{obs} to H_{theor}; and phase lag κ is also calculated by taking the difference of ζ_{obs} and ζ_{theor}. The obtained values of G and κ are as shown in Table 5, where the plus sign of κ means that the observed phase is behind the theoretical one.

The values shown in Table 5 are entirely independent of the epochs of obser-

Table 5.

	M_2		S_2		O_1		K_1	
	$G \equiv \frac{1-3k/2}{+h}$	κ	$G \equiv \frac{1-3k/2}{+h}$	κ	$G \equiv \frac{1-3k/2}{+h}$	κ	$G \equiv \frac{1-3k/2}{+h}$	κ
I	1.18±0.02	+1°.96	1.16±0.03	−3°.33	1.15±0.04	−1°.93	1.29±0.07	+22°.73
II	1.21±0.02	+1°.20	1.23±0.02	+2°.22	1.12±0.05	−5°.70	1.31±0.13	+ 0°.95
III	1.23±0.02	+2°.42	1.23±0.02	+5°.37	0.99±0.05	−0°.39	1.34±0.10	+19°.62

vations and the latitudes of observation stations; consequently the difference of those values, if it has any significance, is interpreted as showing local character of tidal factor. But, in this point, the reliability of numerical values for each component tide should carefully be examined. From the fact that those values of S_2–, O_1–, and K_1–components in each station are largely diverse from each other, but those of the M_2–component show comparatively good accordance for the three stations, it is clear that the diversity with respect to the S_2–, O_1–, and K_1–components is explained by the incompleteness of extraction of the component tides in the process of analysis, which will be improved by elongation of the observation period. Hence the following is generally concluded. "Concerning all the component tides except M_2, the reliable values of tidal factor of gravity can never be obtained from the observations for a period shorter than one month." For this reason the succeeding discussions will be restricted to the lunar semi-diurnal component alone.

Concerning the appropriate period of observation for M_2–component fit to the treatment of a certain problem, the following consideration may serve the purpose. Gravity values observed during one month at station I were tentatively divided into two periods of a fortnight and four periods of a week, and harmonic analysis was applied to the values for each period. The results are as shown in Table 6. From

Table 6.

	Period of analysis	$G(M_2)$
Monthly	June 12th 00h～July 12th 00h, 1954	1.18
Fortnightly	June 12th 00h～June 26th 12h, 〃	1.17
	June 26th 12h～July 11th 00h, 〃	1.26
Weekly	June 12th 00h～June 19th 06h, 〃	1.25
	June 19th 06h～June 26th 12h, 〃	1.11
	June 26th 12h～July 3rd 18h, 〃	1.60
	July 3rd 18h～July 11th 00h, 〃	0.93

these results the following is generally concluded. "Concerning the M_2–component, a reliable value for the tidal factor of gravity cannot be expected from the observations for a period shorter than a fortnight." Needless to say, the word "reliable" in the above expression is a vague term, but in the present case it is conveniently used to designate a result with an accuracy in numerical values deserving of discussion on the local effect of the station upon the tidal factor of gravity or, simply expressed, the values with a mean error of within 0.02 of the tidal factor of gravity. From the above-mentioned results it is understood that one month is the necessary minimum period for obtaining a reliable tidal factor of the M_2–component; and the discussion

on the problem whether one month is a sufficient period or not for the detailed study on local character of the tidal factor of gravity will be made in a later paragraph.

The three stations I, II and III were primarily selected for the convenience of detecting the local character, if existing, of the tidal factor of gravity, the three stations being representative in the point of effects concerned with crustal structure and ocean tide as seen in Fig. 1. Consequently the values for the M_2-component obtained at the three stations are considered to be highly reliable and deserve the discussion on the problem of the local effect of crustal structure and ocean tide upon the value of the tidal factor of gravity.

4. Local effect on tidal factor of gravity

As the effects which are supposed to affect the value of G, two effects caused by the oceanic tidal change and the local geological or crustal structure are most conspicuous; but the influence of height difference of the observation stations will be worthy of consideration. In the following the orders of magnitude of influence upon G by these causes will be estimated.

(1) *Effect caused by the height difference of observation station*

As already described in section 1, the tide-producing potential W_2 is a function of r, in other words, a function of height of observation station; consequently the vertical component of tidal force depends also on height. Differentiating W_2 with respect to r, we get

$$\frac{\partial W_2}{\partial r}=\gamma MD^{-3}r(3\cos^2\theta-1)\,,$$

which represents that the vertical component of tidal force is proportional to r. In the present case the maximum height difference between three observation stations is only 40 m; and the effect due to this height difference calculated by the above equation is estimated to be 1/160000 of the original force. With regard to Love numbers, they are also functions of r, but the effect of height difference upon them is negligibly small in the similar order of the above case. Hence, the effect on the value of G by the height difference between the observation stations may safely be disregarded in the present case.

(2) *Effect caused by the ocean tide*

The suppcsed effects upon the value of G caused by the ocean tide are generally divided into three principal parts. The first is the effect of the vertical component of attraction by the tidal change of sea water, the second the effect of the vertical displacement of the observation point caused by the tidal load of sea water, and the third the effect caused by the distortion of potential field as the result of deformation

of the ground and sea bottom. Besides these, the effect originating from the horizontal displacement of the ground caused by the tidal load of sea water may also to be taken into account, but this effect was ascertained to be negligibly small compared with the above described three effects in the present case. On the effects caused by the tidal change of sea water the station to be discussed is the station III, Shiono-misaki, whose observation point is 74 m high above the sea level and 400 m distant from the sea shore of Pacific Ocean. Concerning other two stations, I and II, the effects concerned with the ocean tide are ascertained to be negligibly small as the distances from the nearest sea shore of Pacific Ocean are 110 km and 140 km respectively.

The vertical component of attraction of M_2-period caused by the neighbouring sea water for the observation point at the station III is calculated as nearly 0.9μ gal $\cos(2nt-165°)$. Concerning the effect originated from the vertical displacement of ground at the observation point caused by the tidal load of sea water, the formula of H. Nagaoka (**7**) on the elastic deformation of the ground by surface load is applicable. According to his formula the effect caused by the vertical displacement of ground is computed to be more than 2.3μ gal $\cos(2nt-165°)$, and this value seems to be too large from the practical point of view. This improbable large value seems to originate from the fact that the real elastic properties of the earth's crust is far remote from the one which is assumed for the formation of Nagaoka's formula. Consequently some theory except Nagaoka's formula must be adopted for our case. This problem will be discussed together with the problem concerning the distortion of potential field caused by the deformation of the ground and sea bottom in the following.

The effect caused by the distortion of potential field as the result of the deformation of the ground and sea bottom is considered to be important part in calculation of oceanic tidal effect, but its strict estimation is very difficult and is beyond our ability at present. But fortunately, the theory of H. Takeuchi (**8**) on the deformation of the earth by surface loads, which is originally available for the case that the surface loads can be expressed by a spherical harmonic, can be approximately applied in present case. According to his theory, the total effects upon gravity change caused by the tidal change of sea water within a distance of several hundreds kilometres from the observation point are calculated to be nearly 1.1 times the attraction term. In present case the attraction term is calculated to be 0.9μ gal $\cos(2nt-165°)$. Accordingly the total effects of the ocean tide are computed to be nearly 1.0μ gal $\cos(2nt-165°)$.

The value obtained by subtracting the above computed effects from the value of M_2-component at the station III in Table 3 is considered to be completely free from

the influences of ocean tide. In this way,

$$64.0\mu\,\text{gal}\,\cos(2nt-182°.42)-1.0\mu\,\text{gal}\,\cos(2nt-165°)$$
$$=63.0\mu\,\text{gal}\,\cos(2nt-182°.73)\,.$$

Since the corresponding theoretical value is

$$52.2\mu\,\text{gal}\,\cos(2nt-180°.00)\,,$$

the values of G and κ are calculated as follows:

$$G=1.21 \qquad \kappa=2°.73\,.$$

Consequently, the values of G and κ for the three stations which are considered to be entirely free from the effects of ocean tide are as shown in Table 7.

Table 7.

	Station	G (M_2)	κ (M_2)	g (gal)	Bouguer anomaly
I	Kyoto	1.18±0.02	+2°.0	979.722	−12 mgal
II	Chikubushima	1.21±0.02	+1°.2	979.714	−45 〃
III	Shionomisaki	1.21±0.02	+2°.7	979.740	+139 〃

(3) *Effect caused by the local crustal structure*

The values of G listed in Table 7 are considered to be ones corrected for all influences but the local geological or crustal structure. Concerning the local effect caused by the geological, or generally, crustal structure upon the value of G, its exact theoretical treatment is supposed to be very difficult and laborious; but the problem is very important and attractive for the studies of not only the earth tide but also the nature of the earth's crust, and it shall be rigorously treated in the near future. For the advancement of study on this problem, first of all, it is a most necessary and pressing matter to raise the accuracy of measurement by using more highly sensitive gravimeters and by observing for periods much longer than one month, and, on the other hand, to increase the number of observation stations under different conditions to better examine the various effects. Under these considerations, the highly precise determination of the G-value at two stations by observations during one year with two evacuated gravity variometers of especially high sensitivity in nearly 0.0001 mgal/mm/5 m, and the concurrent observations at twenty stations in our country with twenty gravity variometers of sensitivity in nearly 0.01 mgal/mm/2 m are scheduled to be put into effect in this year. All gravity variometers, now in construction, are of the type of double bifilar suspension which are described in detail in part I of the present series of investigations (**3**). The detailed report on the resuls of these observations will be made on another occasion.

In the present case the G-value at station I, Kyoto, of 1.18 will provisionally be adopted as the representative value, because the crustal thickness at Kyoto is estimated to be nearly 30 km from the seismometric investigation, the value of which being considered to be nearly a mean thickness in the world over.

5. Love numbers and tidal upheaval of the ground

The values of G obtained from the present investigation and D obtained from the tiltmetric observation by E. Nishimura (2) are as follows:

$$G=1-\frac{3}{2}k+h=1.18\,,$$

$$D=1+k-h=0.66\,.$$

By combining the both values, Love numbers h and k are calculated as follows:

$$h=0.66\,,\qquad k=0.32\,.$$

Applying the value of h above calculated to the expression of the radial displacement $U=h\frac{W_2}{g}$ at the observation station of Kyoto, the tidal upheaval of the ground is estimated as follows:

Range of M_2-component in tidal upheavel of the ground at Kyoto $=21.3$ cm,

Range of lunisolar total components in tidal upheaval of the ground at Kyoto $=50.5$ cm.

And the greatest tidal upheaval of the ground that could occur on the earth is calculated as follows:

Range of greatest tidal upheaval of the ground on the earth $=51.5$ cm.

6. Summary

The purpose of the present investigation was to explain the diversity in values of the tidal factor of gravity obtained by many observers over the world. The gravimetric observations for earth tide were made under consideration of the problems of instrumental errors, reliability of instrumental scale value, length of observation period, etc. on the one hand, and the problem of effects caused by the ocean tide and local geological or crustal structure on the other. From the present one month's observations made at three stations of different crustal structure and oceanic tidal effect, it is generally concluded that the values of G obtained at the three stations are highly trustworthy, at least for the M_2-component. And it was roughly estimated that the small difference of the third figures in G-values is mainly attributed to the effects caused by the ocean tide and local crustal structure. And, at the present stage, the most reliable value of G was calculated to be 1.18 ± 0.02, and the greatest tidal upheaval of the ground on the earth was estimated as nearly 52 cm.

For the further advancement of study on the present problem, the following researches are earnestly recommended, and some of them are scheduled to be made at our institute at an early opportunity: gravimetric observations of the earth tide with more highly sensitive gravity variometer of continuous self-recording as compared with the usual gravimeter of sensitivity in the order of 0.1 mgal/div. and intermittent reading; concurrent observations with the gravity variometers of same type and same sensitivity at many stations where the conditions for the effects by the ocean tide, crustal structure and others are largely different, for example, at widely different latitudes, at the middle of the continent, at the continental margin, and at the solitary island in the ocean. From these precise and complete measurements of tidal variation of gravity, the mode and behaviour of the earth and earth's crust against the tidal forces will be revealed. And it will afford us an important and powerful tool to better understand the nature of the earth and the earth's crust.

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Observation I

1954-VI-10	11	12	13	14	15	16	17	18	19
h m									
0.00	616.5	619.6	622.6	625.7$_5$	629.0	632.3	635.6	638.9	642.3
30	16.6	19.7	22.8	25.8	29.1	32.4	35.6	39.0	42.3
1.00	16.6	19.8	22.9	25.9	29.3	32.5	35.7	39.0	42.3$_5$
30	16.6	19.9	23.0	26.1	29.4	32.6	35.8	39.1	42.3$_5$
2.00	16.8	19.9	23.1	26.3	29.5	32.8	35.9	39.2	42.4
30	16.8	19.9$_5$	23.2	26.4	29.6	32.9	36.0	39.3	42.4
3.00	16.8	20.0	23.2	26.4$_5$	29.7	33.0	36.2	39.5	42.6
30	16.8	20.0	23.2	26.5	29.8	33.1	36.4	39.6	42.8
4.00	16.8	20.0	23.1	26.6	29.9	33.1	36.5	39.7	43.0
30	16.8	20.0	23.1	26.6	30.0	33.1	36.5	39.8	43.1
5.00	16.8	20.0	23.1	26.6	30.0	33.2	36.4	39.9	43.2
30	16.8	20.0	23.2	26.6	30.0	33.3	36.4	40.0	43.2$_5$
6.00	16.7	20.0	23.2	26.5	30.0	33.3	36.7	40.2	43.3
30	16.7	19.8	23.2	26.5	29.9	33.3	36.6	40.3	43.4
7.00	16.7	19.7	23.0	26.3$_5$	29.9	33.2	36.5	40.1	43.4
30	16.7	19.7	23.0	26.3	29.9	33.1$_5$	36.5	40.1	43.4
8.00	16.7	19.7	23.0	26.3	29.8	33.1	36.5	40.0	43.4
30	16.7	19.7	23.0	26.2	29.8	33.0	36.4	40.0	43.4
9.00	16.7	19.7	23.0	26.1	29.8	32.9	36.3	39.8	43.4
30	16.8	19.8	22.9	26.1	29.5	32.8	36.3	39.8	43.3
10.00	16.9	19.8	22.9	26.1	29.4	32.7	36.1$_5$	39.7	43.2
30	17.0	19.9	23.0	26.2	29.4$_5$	32.7	36.1	39.6	43.1
11.00	17.2	20.0	23.1	26.3	29.5	32.7	36.0$_5$	39.5	43.0
30	17.3$_5$	20.2	23.1$_5$	26.4	29.5	32.8	36.1	39.6	42.9
12.00	615.1 17.5	20.4	23.2	26.5	29.7	32.9	36.1	39.6	42.8
30	15.0 17.6	20.5	23.3	26.6	29.8	33.0	36.2	39.6	42.8
13.00	15.0 17.7	20.6	23.6	26.8	29.9	33.1	36.3	39.6	42.8$_5$
30	15.0 17.8	20.9	23.8	27.0	30.1	33.3	36.5	39.7	42.8
14.00	15.1 17.9	20.9	23.9	27.2	30.2	33.5	36.6$_5$	39.8	43.0
30	15.2 18.0	21.0	24.1	27.5	30.5	33.7	36.8	40.0	43.1
15.00	15.3 18.1	21.3	24.3	27.7	30.9	33.9	36.9$_5$	40.2	43.3
30	15.3 18.3	21.4	24.4	27.8	31.1	34.1	37.1	40.4	43.5
16.00	15.4 18.3	21.5	24.5	27.9	31.3$_5$	34.3	37.4	40.5	43.7
30	15.5 18.4	21.6	24.8	28.0$_5$	31.4	34.5	37.6	40.7	43.9
17.00	15.5 18.4$_5$	21.7	24.9	28.2	31.6	34.5$_5$	37.8	41.0	44.1
30	15.5 18.5	21.7	25.1	28.4	31.7	34.7	38.0	41.2$_5$	44.3
18.00	15.5 18.5$_5$	21.8	25.1	28.4	31.9	34.9	38.2	41.4	44.5
30	15.5 18.6	21.8	25.1	28.4	32.0	35.0	38.3	41.5$_5$	44.6
19.00	15.6 18.6	21.9	25.3	28.5	32.0	35.1	38.5	41.7	44.8
30	15.6 18.6$_5$	21.9	25.3	28.6	32.1	35.2	38.6	41.8	45.0
20.00	15.7 18.7	22.0	25.3	28.6	32.1	35.3	38.6	41.9$_5$	45.1
30	15.7 18.7$_5$	22.0	25.3	28.6	32.1	35.4	38.6	42.0	45.2
21.00	15.8 18.8	22.0	25.3	28.7	32.1$_5$	35.4	38.6	42.0$_5$	45.3
30	15.8 19.0	22.0	25.3	28.7	32.2	35.4	38.7	42.1	45.3
22.00	16.0 19.1	22.1	25.4	28.8	32.2	35.3	38.7	42.2	45.3$_5$
30	16.1 19.3	22.1	25.4	28.8$_5$	32.2	35.4	38.7	42.2	45.4
23.00	16.2 19.4	22.4$_5$	25.6	28.9	32.2	35.5	38.8	42.2	45.3
30	16.5 19.4	22.5	25.7	29.0	32.3	35.6	38.8	42.2$_5$	45.3

The numerals express the dial scale of the gravimeter used, and one scale division corresponds to 0.18473 mgal in gravity change.

Observation I (continued)

20	21	22	23	24	25	26	27	28	29	30	VII-1
645.3	648.8	651.4	654.5	657.8	663.1	666.3	669.6	673.2	676.3	679.1_5	682.0
45.4	48.8	51.4	54.6	57.9	63.2	66.4	69.7	73.3	76.4	79.1_5	82.0
45.4	48.9	51.3	54.6	57.9	63.3	66.6	69.9	73.4	76.5	79.3	82.1
45.5	48.9	51.3	54.6	57.8	63.3	66.7	69.9	73.5	76.7	79.4	82.2
45.5	48.9_5	51.3	54.5	57.8	63.3	66.7	70.0	73.7	76.8	79.5	82.3_5
45.6	49.0	51.3	54.5	57.8	63.3	66.7	70.2	73.8	76.9	79.6	82.5
45.7	49.0	51.4	54.4_5	57.7	63.3	66.7	70.3	73.8_5	77.0	79.7	82.6
45.8	49.0_5	51.4	54.5	57.7	63.3	66.6_5	70.3	73.9	77.1	79.9	82.6_5
45.8	49.2	51.4	54.5	57.8	63.3	66.6	70.3	73.9	77.1	80.0	82.8
45.9	49.3	51.5	54.5_5	57.8	63.2	66.6_5	70.2	73.9	77.2	80.0	82.9
45.9	49.4	51.6	54.6	57.8	63.2	66.7	70.0	73.8	77.2	80.0_5	83.0
46.0	49.5	51.6	54.7	57.8	63.1	66.6_5	70.0	73.7	77.2	80.0_5	83.1
46.4	49.6	51.7	54.7	57.8	63.1	66.6	69.9	73.6	77.1	80.0	83.1
46.4	49.8	51.8	54.8	57.8	63.1	66.5	69.9	73.5_5	77.0	80.0	83.2
46.4	49.8	52.0	54.9	57.9	63.1	66.4_5	69.9	73.5	76.8	79.8_5	83.2
46.4	49.8	52.1	55.0_5	58.0	63.1	66.4_5	69.8	73.4	76.7	79.7	83.2
46.6	49.9	52.2	55.2	58.0	63.1	66.4	69.8	73.3	76.5_5	79.6	83.2_5
46.6	50.0	52.3	55.3	58.1	63.2	66.4_5	69.6	73.2	76.5	79.5	82.7
46.5	—	52.4	55.4	58.3	63.4	66.5	69.6	73.2	76.4	79.4_5	82.7
46.5	—	52.3	55.5	58.5	63.5	66.6	69.7	73.1_5	76.4	79.3	82.6
46.5	—	52.3	55.6_5	58.7	63.6_5	66.7	69.8_5	73.2	76.4	79.2	82.5
46.4	—	52.3	55.7	58.7	63.8	66.9	70.0	73.3	76.4	79.2	82.4
46.3_5	*49.0	52.4	55.8	58.7	63.9	67.0	70.1_5	73.5	76.4_5	79.3	82.4
46.3_5	49.1	52.4	55.8	58.9	64.1	67.1	70.3_5	73.7	76.6	79.3	82.4_5
46.3	49.0	52.4	55.9	59.1	64.3	67.3	70.5	73.8	76.7	79.4	82.4
46.3	49.0	52.4	55.9	59.2	64.4	67.5	70.7	74.0	76.8_5	79.5	82.4_5
46.2	49.1	52.4	55.9	59.3	64.5	67.8	70.9	74.2	77.0	79.6	82.6
46.2	49.1	52.4	55.8	59.3_5	64.6_5	67.9	71.1_5	74.4	77.2	79.7	82.6_5
46.2	49.1_5	52.4	55.8	59.4	64.7	68.1	71.3_5	74.6	77.4	79.9	82.8
46.3_5	49.2	52.4	55.8	$*61.1_5$	64.8_5	68.3	71.6	74.8	77.6	80.2	83.0
46.5	49.3	52.5	55.8	61.2	64.9	68.4	71.7_5	75.0	77.9	80.4	83.2
46.7	49.4	52.5	55.9	61.2_5	65.0	68.4	71.9	75.2	78.1	80.6	83.4_5
46.8	49.5	52.6	56.0	61.4	65.1	68.5	72.0	75.4	78.3_5	80.9	83.6_5
46.9	49.5	52.7	56.1	61.4	65.1	68.6	72.1	75.6	78.4	81.1	83.8_5
47.1	49.6	52.8	56.2	61.5	65.0	68.6	72.1_5	75.7	78.5	81.2	84.1
47.3	49.8	52.9	56.1	61.6	65.1	68.7	72.3	75.7	78.6	81.4	84.3
47.6	50.0	53.0_5	56.4	61.7_5	65.1_5	68.8	72.4	75.8	78.7	81.5	84.4
47.8	50.0_5	53.2	56.5	61.9	65.2	68.8	72.4	75.9	78.8	81.5_5	84.6
48.0	50.3	53.4	56.6	62.0	65.3	68.9	72.4	76.0	78.9	81.7	84.7
48.2	50.4	53.5	56.6	62.1	65.4	68.9	72.4	76.0	79.0	81.8	84.8
48.3	50.6	53.7	56.9	62.2	65.4_5	68.9	72.5	75.9	79.0_5	81.8	84.8_5
48.4	50.7	53.9	57.0	62.3	65.5	69.0	72.5	75.8	79.0	81.8	84.9
48.5	50.8	54.1	57.1	62.5	65.5_5	69.1	72.5	75.8	79.0	81.9	85.0
48.6	50.9	54.2	57.4	62.6	65.7	69.1	72.6	75.8	78.9	81.9	84.9
48.6	51.0	54.3_5	57.4_5	62.7	65.8	69.1	72.7	75.8	78.8_5	81.9	84.9
48.7	51.2	54.4	57.6	62.9	66.0	69.2	72.7_5	75.9	78.9	81.9	84.8
48.7	51.3	54.4	57.7	63.0	66.1	69.4	72.8	76.0	78.9	81.9	84.9
48.7	51.3	54.5	57.8	63.1	66.3	69.5	73.0	76.2	79.0	81.8_5	84.9

* Where the series of reading was shifted to another series because of hitch in the illuminating lamp attached to the gravimeter.

Observation I (continued)

2	3	4	5	6	7	8	9	10	11	12	13
684.9	687.9	690.4	693.7	697.3	700.8	704.1	706.9	710.2$_5$	713.4	716.4	719.6
84.7$_5$	87.9	90.4	93.7	97.3	00.8	04.1	07.0	10.3	13.4	16.5	19.9
84.8$_5$	88.0	90.4	93.7	97.3	00.8	04.1$_5$	07.0	10.3	13.4	16.7	19.9
84.8$_5$	*87.3	90.4	93.8	97.3$_5$	00.7	04.1	07.0	10.3	13.5	16.6	19.9
84.9	87.3	90.5	93.8	97.3$_5$	00.7	04.1$_5$	07.1	10.4	13.5	16.8	20.0
85.0	87.4	90.6	93.9	97.1	00.8	04.1	07.1	10.4	13.5	16.9	19.9$_5$
85.0	87.6	90.7	93.9	97.3	00.8	04.1	07.1	10.4	13.5$_5$	16.8	20.0
85.5	87.7	90.8	93.9	97.3	00.8	04.1$_5$	07.1	10.3	13.7	16.8	20.0
85.6	87.7	90.9	94.0	97.3$_5$	00.8	04.1$_5$	07.1	10.3	13.5	16.9	20.4
85.6$_5$	87.9	91.1	94.2	97.5	00.9	04.1	07.0$_5$	10.3	13.5	17.0	20.6
85.8$_5$	88.1	91.2	94.3$_5$	97.5	01.0	04.1$_5$	07.0	10.3	13.5	17.0	20.4
85.9	88.3	91.3	94.4$_5$	97.5	01.0	04.1$_5$	07.0	10.3	13.5	16.9	20.4
85.8$_5$	88.4$_5$	91.4	94.8	97.7	01.1	04.2	07.0	10.3	13.5	16.8	20.5
86.0	88.5	91.5	94.9	97.8	01.2	04.3	07.2	10.3	13.4	16.8	20.5
86.0$_5$	88.5	91.6	95.0	97.9	01.3	04.4	07.2	10.3$_5$	13.4	16.6	20.4$_5$
85.9	88.5	91.7	95.2	98.1	01.4	04.5	07.2$_5$	10.4	13.4	16.6	20.3$_5$
85.9	88.5	—	95.3	98.2	01.5	04.6	07.4	10.4$_5$	13.5	16.6	20.3$_5$
86.0	88.4	91.7	95.3	98.3	01.6	04.7	07.5	10.5	13.4	16.7	20.2$_5$
85.7$_5$	88.4	91.7	95.4	98.3$_5$	01.6$_5$	*04.0	07.6$_5$	10.6	13.5	16.7	20.2$_5$
85.7	88.3$_5$	91.5	95.4	98.5	01.7$_5$	04.2	07.7	10.6	13.6	16.7	20.2$_5$
85.7	88.3	92.0	95.4	98.6	01.8	04.3$_5$	07.8	10.6	13.8	16.7$_5$	20.2$_5$
85.5	88.2	91.8	95.3	98.6$_5$	01.9	04.5	07.9	10.7	13.9	16.8	20.3
85.4$_5$	88.1	91.9	95.3	98.7	01.9	04.6	08.0	10.9	14.0	15.9	20.5
85.4$_5$	88.0$_5$	91.9	95.3	98.7	01.8$_5$	04.7	08.1	11.0	14.1	17.1	20.5
—	88.0	91.9	95.3	98.7	01.9	04.8	08.2$_5$	11.2	14.2	17.2	20.6
85.3	88.0	91.8	95.3	98.7	02.1	04.9	08.4	11.3	14.4	17.3	20.7
85.4	88.0	91.9	95.3	98.6$_5$	02.1	05.0	08.5	11.5	14.6	17.4	20.8
85.5$_5$	88.0	91.8	95.3	98.6	02.1	05.0	08.6	11.6	14.8	17.7	21.0
85.7	88.0$_5$	91.8	95.4	98.6$_5$	02.2	05.0	08.7	11.7	14.9	17.8	21.3
85.8	88.1$_5$	91.6	95.4	98.7	02.3	05.1	08.8	11.9	15.1	17.9	21.6
85.9$_5$	88.2	92.0	95.4	98.8	02.3	05.1$_5$	08.8	12.0	15.2	18.0	21.9
86.2	88.4	92.0	95.5	98.8	02.3	05.3	08.9	12.0$_5$	15.3	18.1	22.1
86.5	88.6	92.0	95.6	98.9	02.4	05.3	09.0	12.1	15.5	18.3	22.3
86.7	88.7	92.3	95.7	98.9	02.4	05.4	09.1	12.2	15.5	18.4	22.4
86.8	89.0	92.3	95.8	99.1	02.7	05.5	09.2	12.2	15.6	18.6	22.6
87.0	89.2	92.6	95.9	99.2	02.7$_5$	05.5	09.1$_5$	12.3	15.3	18.8	
87.2	89.5	92.8	96.1	99.3	02.8	05.6	09.1$_5$	12.4	15.1$_5$	19.0	
87.4	89.7	92.9	96.3	99.5	03.0	05.6$_5$	09.4	12.4	15.6	19.1	
87.6	90.0	93.2	96.5	99.8	03.1	05.5$_5$	09.5	12.4	15.8	19.2	
87.7$_5$	90.1	93.4	96.7	700.0	03.2	05.9	09.5$_5$	12.5	15.9	19.2	
87.8	90.2	93.5	96.8	00.1$_5$	03.3	06.1	09.5$_5$	12.6	15.7$_5$	19.2	
87.9	90.3	93.5	96.9	00.3	03.4	06.3	09.6	12.7	15.6$_5$	19.2$_5$	
87.9	90.3	93.6	97.0	00.4	03.6	06.4	09.7	12.7$_5$	15.7	19.3	
87.9	90.4	93.7	97.1	00.5$_5$	03.7	06.6	09.8	12.8	15.8$_5$	19.3$_5$	
88.0	90.4	93.7	97.2	00.7	03.8	06.7	10.0	12.9	15.9	19.4	
88.0	90.4	93.7	97.3	00.7	03.8	06.8	10.0	13.0	16.1	19.5	
88.0	90.4	93.7	97.3	00.7	03.9	06.8	10.0	13.2	16.4	19.5	
87.9	90.4	93.7	97.4	00.8	04.0	06.9	10.1	13.4	16.4	19.5	

Observation II

1955-IX-28		29	30	X-1	2	3	4	5	6	7	8
h m											
0.00		064.3	068.4	073.1	077.9	082.3	088.5	092.7	097.9	104.0_5	110.1_5
30		64.4	68.6	73.2	78.0	82.6	88.4	92.7	98.0	04.1	10.2
1.00		64.6	68.8	73.5	78.2	82.9	88.4	92.7	98.0	04.1	10.3
30		64.7	69.0	73.8	78.3	83.1	88.5	92.8	98.0	04.2	10.3
2.00		64.8	69.3	74.0	78.5	83.3	88.6	92.8	98.0	04.3	10.4
30		64.8	69.5	74.2	78.8	83.5	88.8	92.9	98.1	04.4	10.4
3.00		65.0	69.7	74.5	79.1	83.8	89.0	93.1	98.2	04.5	10.5
30		65.2	69.9	74.7	79.3	84.0	89.3	93.3	98.3	04.6_5	10.6
4.00		65.3	70.0	74.9	79.5	84.3	89.6	93.6	98.4	04.7_5	10.7
30		65.4	70.1	75.0	79.8	84.6	89.9	93.8	98.5	04.9	10.8
5.00		65.5	70.2	75.1	80.1	85.0	90.1	94.0	99.1	05.1_5	11.0
30		65.5	70.3	75.2	80.4	85.2	90.3	94.3	99.6	05.3_5	11.3_5
6.00		65.5	70.3	75.4	80.5	85.2	90.5	94.5	99.7	05.6	11.6
30		65.5	70.3	75.4	80.5	85.7	90.7	94.9	99.9	06.0	11.8
7.00		65.6	70.3	75.4	80.5	85.7	90.8	95 1	100.1	06.3	11.9_5
30		65.6	70.3	75.4	80.5	85.7	90.9	95.3	00.3_5	06.5	12.2
8.00		65.6	70.3	75.4	80.5	85.8	91.0	95.5	00.5	06.7	12.4_5
30		65.6	70.3	75.4	80.5	85.9	91.0	95.7	00.8	07.0	12.7
9.00		65.6	70.3	75.4	80.5	85.9	91.0	95.8	00.9	07.3	13.0_5
30		65.6	70.3	75.4	80.5	85.9	91.0	95.8	00.9_5	07.5	13.3
10.00		65.6	70.4	75.4	80.5	85.9	91.0	95.8_5	01.0_5	07.6	13.5
30		65.7	70.4	75.4	80.5	86.0	91.0	95.8_5	01.3	07.7	13.6_5
11.00		65.8	70.4	75.4	80.5	86.0	91.1	95.8	01.4	07.8_5	13.8
30		65.9	70.5	75.4	80.6	86.0	91.1	95.9	01.5	07.9_5	14.1
12.00		66.1	70.9	75.6	80.6	86.1	91.1	96.0	01.6	07.9_5	14.2_5
30		66.3	71.2	75.7_5	80.7	86.1	91.3	96.1	01.7	08.1_5	14.3
13.00		66.5	71.5	75.9	80.9	86.3	91.4	96.1	01.8	08.2	14.4
30		66.7	71.5	76.2	*80.8	86.5	91.6	96.0	01.8	08.3	14.5
14.00		66.9	71.5	76.4	81.0	86.7	91.7	96.1	02.0	08.4	14.6_5
30		67.1	71.7	76.5_5	81.2	86.9	91.8	96.3	*02.1	08.5	14.7_5
15.00	063.0	67.3	72.0	76.7	81.4	87.1	92.0	96.5	02.3	08.6_5	15.0
30	63.3	67.5	72.2	76.9	81.6	87.3	92.0_5	96.7	02.3	08.6_5	15.1_5
16.00	63.4	67.7	72.4	77.1	81.8	87.6	92.3	96.9	02.5	08.6	15.4
30	63.3	67.8	72.6	77.3	82.0	87.8	92.6	96.9_5	02.6	08.6_5	15.7
17.00	63.4	67.9	72.6	77.4	82.3	88.0	92.6_5	97.1	02.8_5	08.8_5	15.8
30	63.5	67.9	72.8	77.5	82.5	88.2	92.8	97 2	03.0_5	09.0	15.9
18.00	63.5	68.0	72.9	77.6	82.6	88.3	93.0	97.3_5	03.4	09.0	16.4
30	63.4	68.0	72.9	77.7	82.6	88.5	92.9_5	97.7	03.5_5	09.0_5	16.5
19.00	63.2	68.0	72.8	77.7	82.6	88.6	92.9_5	97.8	03.5_5	09.1	16.7
30	63.2	68.0	72.8	77.7	82.6	88.6	92.9_5	97.9	03.6	09.3	16.9
20.00	63.4	68.0	72.8	77.7	82.6	88.6	92.9	98.0	03.6_5	09.5	17.1
30	63.4	68.0	72.8	77.7	82.6	88.6	92.9	98.0_5	03.6_5	09.6	17.3
21.00	63.5	68.0	72.8	77.6	82.6	88.6	92.9	98.0_5	03.7	09.7	17.4
30	63.6	68.0	72.8	77.5	82.6	88.5	92.9	98.0_5	03.7_5	09.8	17.5_5
22.00	63.7	68.0	72.8	77.5	82.5	88.5	92.9	98.0_5	03.8	09.8_5	17.8
30	63.8	68.0	72.8	77.5	82.5	88.5	92.9	98.0	03.9	09.9	18.0
23.00	64.0	68.2	72.9	77.7	82.4	88.5	92.8	97.9	04.0	10.0	18.2
30	64.2	68.5	72.9	77.7	82.4	88.5	92.7	97.9	04.0	10.1	18.3

Observation II (continued)

9	10	11	12	13	14	15	16	17	18	19
118.5	124.3	128.6$_5$	133.2$_5$	139.1	143.8	147.8$_5$	152.2	156.8$_5$	161.8	166.7$_5$
18.6	24.3$_5$	28.8	33.6$_5$	39.2	44.0	48.1	52.3$_5$	56.9$_5$	61.9$_5$	66.8
18.7	24.4$_5$	28.9	33.9$_5$	39.4	44.2	48.3	52.6	57.1	62.1$_5$	66.9
18.9	24.5$_5$	29.0$_5$	34.1	39.7	44.4	48.5	52.9	57.2$_5$	62.3	67.0
19.0	24.6	29.2$_5$	34.3	40.0	44.6	48.7$_5$	53.1	57.5	62.3$_5$	67.2
19.1	24.7	29.3$_5$	34.5	40.2$_5$	44.9	49.0	53.2	57.8	62.6	67.3$_5$
19.2	24.7	29.3$_5$	34.6	40.4	45.1	49.3	53.5	58.0	62.9	67.6
19.3	24.7$_5$	29.4	34.7	40.6$_5$	45.3$_5$	49.5	53.8	58.3$_5$	63.0	67.8
19.4	24.8	29.4	34.8	40.9	45.4	49.6	54.0	58.5	63.2	68.1
19.4$_5$	24.8	29.4$_5$	34.8	41.0	45.5$_5$	49.7	54.4	58.8	63.5	68.3
19.5	24.8$_5$	29 6	34.9	41.0$_5$	45.6	49.8	54.5	59.0	63.6	68.5
19.7	24.9	29.6	34.9$_5$	41.1	45.7	49.9$_5$	54.5	59.2	63.8	68.7$_5$
19.8$_5$	25.0	29.6	35.0	41.2	45.8	49.9$_5$	54.6	59.4$_5$	64.0	68.9
19.9	25.0	29.6	35.0$_5$	41.4	45.8$_5$	49.9$_5$	54.8	59.6	64.3$_5$	69.0$_5$
20.0$_5$	25.1	29.6	35.1	41.4$_5$	45.8$_5$	50.1	55.0	59.6	64.5	69.2$_5$
20.2	25.2$_5$	29.7	35.2	41.4	45.5$_5$	50.1	54.9$_5$	59.8	64.6	69.4
20.4	25.3	29.8	35.3	41.4	45.7	50.0$_5$	55.0	59.8	64.6	69.5
20.6	25.4	29.9	35.4$_5$	41.4$_5$	45.7	50.0$_5$	54.9	59.9	64.7	69.7
20.8$_5$	25.5	29.9$_5$	35.5$_5$	41.5	45.6	50.0$_5$	54.9	59.9	64.7	69.6
21.0$_5$	25.7	30.1	35.6	41.5	45.5	49.9$_5$	54.9	59.9	64.7$_5$	69.6
21.2	25.8	30.2$_5$	35.7	41.5	45.5$_5$	50.0	54.9	59.9	64.9	69.7
21.4	25.9$_5$	30.4	35.8	41.6$_5$	45.6	50.0	54.8	59.8	64.9	69.8
21.5$_5$	26.0$_5$	30.5$_5$	36.0	41.8	45.7	50.2	54.9	59.9$_5$	64.8	69.8
21.7	26.2	30.8	36.3	41.9	45.9$_5$	50.1	55.0	59.9$_5$	64.9	69.9
22.3	26.4	31.0	36.5	42.1	46.1	50.4	54.9	60.0	65.0	69.9
22.4	26.6$_5$	31.2	36.6	42.2$_5$	46.2$_5$	50.5	55.1	60.0	65.0$_5$	70.0
22.5	26.9	31.4	36.9	42.4$_5$	46.3	50.8	55.3	60.2	65.2	70.1$_5$
22.6	27.0	31.5$_5$	37.1	42.7	46.8	50.8$_5$	55.5	60.4	65.3	70.3
22.6	27.2	31.7$_5$	37.2$_5$	42.9	47.0	51.1	55.7$_5$	*60.9	65.3	70.4
22.7	27.3	31.9	37.5$_5$	43.0$_5$	47.1	51.4	55.8	61.1$_5$	65.5	70.5
22.8	27.4	32.1	37.6$_5$	43.2$_5$	47.3	51.5	56.0	61.4	65.5	70.6
22.8	27.5	32.1$_5$	37.8	43.4	47.2$_5$	51.6$_5$	56.2	61.6	65.8	70.8
22.8$_5$	27.5$_5$	32.2$_5$	37.9	43.5$_5$	47.5	51.8$_5$	56.5	61.8	66.0	71.0
23.0$_5$	27.6	32.3	37.9$_5$	43.6	47.8	51.8$_5$	56.7	61.9	66.3	71.1
23.1	27.6	32.3	38.0	43.6	47.7	52.0	56.6	61.9	66.4	71.3
23.2	27.7	32.3$_5$	38.1	43.6	47.7	52.1	56.6	61.9$_5$	66.4	71.4$_5$
23.2	27.7	32.3	38.1	43.5$_5$	47.7	52.2	56.7	62.0	66.5	71.5
23.2$_5$	27.8	32.3	38.1$_5$	43.6$_5$	47.7	52.3	56.9	62.2	66.6	71.6$_5$
23.3	27.9	32.3	38.1	43.6	47.7	52.3	56.9$_5$	62.2	66.8	71.8
23.4	27.9	32.4	38.1	43.6$_5$	47.7	52.2	56.9	62.2	66.8	71.8
23.6	28.0	32.4$_5$	38.1$_5$	43.4$_5$	47.7	52.1	56.8$_5$	61.9$_5$	66.8$_5$	71.8
23.6	28.1	32.4$_5$	38.1$_5$	43.4$_5$	47.7	52.0	56.9	62.1$_5$	66.8$_5$	71.8$_5$
23.7	28.1$_5$	32.5	38.1$_5$	43.4$_5$	47.6	51.9	56.9	62.0	66.8$_5$	71.9
23.9	28.2	32.6	38.2	43.4$_5$	47.5$_5$	51.9$_5$	56.9	62.0	66.8$_5$	71.9
24.0	28.2$_5$	32.7	38.3	43.4$_5$	47.5	51.9$_5$	56.8	61.8	66.8$_5$	71.7
24.1	28.3$_5$	32.8	38.3$_5$	43.5	47.4$_5$	51.9$_5$	56.8	61.7	66.8	71.7
24.2	28.5	32.9	38.6	43.6	47.4$_5$	52.1	56.8	62.0	66.7	71.7
24.2	28.5$_5$	33.0$_5$	28.9$_5$	43.7	47.6	52.1	56.8$_5$	61.9	66.7	71.7$_5$

Observation II (continued)

20	21	22	23	24	25	26	27	28	29	30
171.7	176.4	181.6$_5$	187.5	192.6	197.1	201.3	205.3	209.6	214.0	218.3
71.7	76.4$_5$	81.9	87.5	92.6	97.2	01.5	05.6	09.8	14.2	18.5
71.7	76.5	81.9	87.5	92.6	97.2	01.5	05.7	10.0	14.1	18.7
71.8	76.5$_5$	81.8	87.6	92.6$_5$	97.2	01.7	05.7$_5$	10.2$_5$	14.6	18.8
72.0	76.6	81.9$_5$	87.6	92.7	97.3	01.7	05.9	10.5	14.8	19.0
72.2	76.7	82.0$_5$	87.6	92.8	97.3	01.8$_5$	06.0	10.7	15.0	19.2
72.4	76.8	82.2	87.7	92.8$_5$	97.4	01.9$_5$	06.1$_5$	10.8$_5$	15.2	19.5
72.6	76.9	82.2	87.7	92.9	97.5	02.0	06.2$_5$	11.0	15.4	19.6
72.6	77.1	82.5	88.0	92.9$_5$	97.5	02.0	06.3	11.1	15.5	19.8
72.9	77.3$_5$	82.6	88.0	93.0	97.5$_5$	02.0$_5$	06.3	11.2	15.6	20.0
73.0	77.5$_5$	82.7	88.1	93.1	97.6	02.1	06.4	11.2	15.7	20.3
73.2	77.8	82.9$_5$	88.4	93.2	97.7	02.1	06.4	11.2$_5$	15.8	20.4
73.5$_5$	78.0	83.2	88.6	93.2	97.7	02.1	06.4	11.3	15.8	20.6
73.6	78.1	83.4	88.7	93.3	98.0	02.1	06.4	11.2$_5$	15.8	20.8
73.8	78.2$_5$	83.6	88.9	93.4	98.0$_5$	02.2	06.5	11.3	16.0	20.8
73.9	78.5	83.7$_5$	89.0	93.5	98.1	02.3	06.5	11.3	16.0	20.8
74.0	78.7	83.9	89.3	93.7	98.3	02.3	06.6	11.3	16.0	20.8
74.2	78.8	84.1	89.5	93.8	98.5	02.4	06.7	11.3	16.2	20.8
74.3$_5$	79.0	84.3	89.6	93.9	98.5	02.5	06.7	11.4	16.1	20.9$_5$
74.4	79.1	84.4	89.8	94.1	98.6	02.6	06.8	11.4	16.2	21.0
74.5	79.2	84.6	89.9	94.3	98.8	02.8	06.9$_5$	11.4$_5$	16.2	20.9$_5$
74.5$_5$	79.2$_5$	84.7	90.0	94.5	99.0	02.8$_5$	07.1	11.6	16.3	21.0
74.6	79.4	84.8	90.1	94.6	99.1	03.1	07.2	11.8	16.4	21.0$_5$
74.6	79.4$_5$	85.0	90.2	94.7	99.3	03.3	07.3	11.9	16.5	21.1$_5$
74.6	79.5	85.1	90.4	94.8	99.5	03.4	07.4	12.1	16.6	21.3$_5$
74.7	79.6	85.3	90.5	95.0	99.6	03.5	07.5	12.4	16.8	21.6
74.7$_5$	79.7	85.4	90.7	95.3	99.8	03.6	07.8	12.6	17.0	
74.8$_5$	79.7$_5$	85.6	90.7	95.4	99.9	03.8	07.9	12.7	17.1$_5$	
75.0	79.9	85.6	90.7$_5$	95.5	99.9	04.0	08.0	12.7	17.3	
75.1	80.0	85.6	90.8	95.5	200.1	04.1	08.2	13.2	17.4	
75.2	80.1$_5$	85.8	90.9	95.6	00.1	04.1	08.3$_5$	13.3	17.6	
75.2$_5$	80.1$_5$	85.9	91.0	95.6	00.2	04.2	08.4	13.4	17.8$_5$	
75.3	80.2	86.0	91.2	95.8	00.2	04.2	08.6	13.5	17.9	
75.5	80.3	86.1	91.3	95.8	00.2$_5$	04.3	08.7	13.5	18.0	
75.7	80.4	86.2	91.3	95.9	00.2$_5$	04.4	08.8	13.5	18.1	
75.9	80.7	86.3	91.3$_5$	96.0	00.2$_5$	04.4$_5$	08.8	13.6	17.9	
76.0	80.9	86.4	91.5	96.0	00.3	04.4	08.8	13.6	17.9	
76.1	81.1	86.4	91.7	96.0	00.3$_5$	04.5	08.9	13.5	17.9	
76.2$_5$	81.1	86.5	91.7	96.0	00.4	04.5	08.7	13.5$_5$	17.9	
76.4	81.2	86.6	91.9	96.2	00.4$_5$	04.4	08.8	13.5	17.8	
76.4$_5$	81.3	86.8	91.9	96.2	00.4$_5$	04.5	08.9	13.5	17.7$_5$	
76.5	81.3	86.9	92.0	96.3	00.5	04.5	08.9	13.5	17.7	
76.4$_5$	81.5	87.1	92.0	96.3	00.6	04.6	08.9	13.4	17.7	
76.4	81.6	87.2	92.2	96.5	00.7	04.6	09.0	13.4	17.7$_5$	
76.3$_5$	81.6	87.3	92.3	96.6	00.8	04.8	09.1$_5$	13.5	17.8$_5$	
76.3$_5$	81.6	87.3	92.3	96.7	00.9$_5$	04.8$_5$	09.3	13.6	17.8$_5$	
76.4	81.6$_5$	87.4	92.4$_5$	96.9	01.1	04.9	09.4	13.7	17.9$_5$	
76.4	81.6$_5$	87.4	92.5	96.9	01.2	05.2	09.5	13.8	18.1	

Observation III

1955-XI-16 h m		17	18	19	20	21	22	23	24	25
0.00		548.8	553.2	557.9	562.3	567.2$_5$	571.8	576.3	581.0	585.5
30		49.0	53.1	57.9	62.3	67.2	71.8	76.4	81.2	85.8
1.00		49.0	53.3	57.9	62.4	67.2$_5$	71.8$_5$	76.5	81.3	85.9$_5$
30		49.0	53.3	58.0	62.5	67.2$_5$	71.8$_5$	76.5$_5$	81.4	86.1$_5$
2.00		49.2	53.4$_5$	58.1$_5$	62.6	67.3	71.8$_5$	76.6	81.5	86.3
30		49.2	53.6	58.3	62.7	67.4	71.8$_5$	76.7$_5$	81.5$_5$	86.4
3.00		49.3	53.9	58.4	62.8	67.5	71.9	76.7$_5$	81.6	86.5
30		49.6	54.1	58.5	62.9	67.6	71.9	76.8	81.6$_5$	86.5$_5$
4.00		49.8	54.3	58.7	63.0	67.7	71.9$_5$	76.8	81.7	86.6
30		50.2	54.4	58.8	63.1$_5$	67.8	72.0$_5$	76.9	81.7$_5$	86.7
5.00		50.3	54.7	59.1	63.4	67.9$_5$	72.2	76.9$_5$	81.8	86.7$_5$
30		50.5	54.9$_5$	59.3	63.6	68.1	72.3$_5$	77.1	81.8$_5$	86.7$_5$
6.00		50.6$_5$	55.2	59.5$_5$	63.8	68.2$_5$	72.5	77.2	81.9	86.8
30		50.8$_5$	55.3	59.8	64.0	68.4	72.7	77.3	82.0	86.9
7.00		51.0	55.5	60.0	64.2	68.5	72.9	77.4$_5$	82.1	86.9
30		51.0$_5$	55.7	60.2	64.3	68.7$_5$	73.0	77.6	82.2	86.9
8.00		51.2	55.8	60.4	64.4	68.9$_5$	73.2$_5$	77.7$_5$	82.3$_5$	87.0
30		51.3$_5$	55.9$_5$	60.6	64.5	69.1	73.4	77.9$_5$	82.5	87.2
9.00	547.9	51.4	56.0	60.7$_5$	64.7	69.3$_5$	73.6	78.1	82.6$_5$	87.4
30	48.0$_5$	51.3$_5$	56.0	60.8$_5$	64.8	69.5	73.8	78.3	82.8	
10.00	48.2	51.3	55.9$_5$	60.9	64.9	69.6	73.9	78.4	82.9$_5$	*86.6
30	48.2$_5$	51.5	56.2	60.9$_5$	65.1	69.7$_5$	74.0	78.5	83.1$_5$	86.8
11.00	48.3	51.5	56.3	61.0	65.1	69.8$_5$	74.2	78.7	83.3	86.9
30	48.3	51.6$_5$	56.2	61.0	65.1$_5$	69.8$_5$	74.2$_5$	78.9	83.5	87.0
12.00	48.2	51.7	56.2	61.0$_5$	65.2	69.8	74.3	79.1	83.7	87.1
30	48.2$_5$	51.8	56.4	61.0$_5$	65.2$_5$	69.7$_5$	74.4	79.2	83.8	87.4
13.00	48.5	51.9	56.5$_5$	61.0$_5$	65.3$_5$	69.9	74.5	79.3	83.9	87.6
30	48.5	51.9	56.6	61.2	65.4	69.9	74.6	79.3	84.0	87.7
14.00	48.6	51.9$_5$	56.7	61.1	65.4	70.0	74.7	79.3$_5$	84.1	87.8$_5$
30	48.7	52.2	56.8$_5$	61.1$_5$	*66.0	70.0	74.8	79.4	84.1$_5$	87.9$_5$
15.00	48.9	52.3	57.0	61.4	66.1$_5$	70.1	74.9	79.4$_5$	84.2	88.0
30	48.9$_5$	52.5	57.2	61.4	66.2	70.2	74.9$_5$	79.5	84.2	88.1
16.00	48.9$_5$	52.7	57.3	61.5	66.2	70.4	75.0	79.5	84.2	88.1$_5$
30	49.1	52.8	57.5	61.6$_5$	66.2$_5$	70.4	75.0$_5$	79.5	84.3	88.2
17.00	49.1	52.9	57.6$_5$	61.7	66.3	70.6	75.1	79.5$_5$	84.3	88.2
30	49.2	53.0	57.8	62.0$_5$	66.4$_5$	70.7	75.2	79.6	84.3$_5$	88.2
18.00	49.3	53.1	57.9	62.1$_5$	66.5$_5$	70.8	75.2$_5$	79.7	84.4	88.2
30	49.3	53.3	58.0	62.3	66.7	71.0	75.3	79.8	84.4	88.2$_5$
19.00	49.3	53.4	58.0	62.3	66.8	71.1	75.3$_5$	79.9	84.4$_5$	88.3
30	49.3	53.5	58.0	62.2	66.9	71.2	75.4	79.9	84.4$_5$	88.3$_5$
20.00	49.2	53.5	58.1	62.2$_5$	67.1	71.3	75.5	79.9	84.5	88.4$_5$
30	49.2	53.5	58.1	62.3	67.1	71.4	75.6	80.0	84.5	88.5
21.00	49.0	53.4	58.1	62.3	67.2	71.5	75.7	80.1	84.6	88.6
30	49.0	53.4	58.1	62.4	67.2	71.6	75.8$_5$	80.2	84.7	88.7
22.00	48.9	53.3	58.0$_5$	62.3$_5$	67.2$_5$	71.7	76.0	80.5	84.8	88.8
30	48.9	53.3	58.0$_5$	62.4$_5$	67.1$_5$	71.7$_5$	76.1	80.6	84.9	88.9$_5$
23.00	48.9	53.2	58.0	62.4$_5$	67.2	71.8	76.1$_5$	80.7	85.1	89.1
30	48.9	53.2	57.9	62.4$_5$	67.2	71.8	76.2	80.9	85.4	89.4

Observation III (continued)

26	27	28	29	30	XII-1	2	3	4	5	6
589.6	593.9$_5$	598.3$_5$	602.7	607.1	611.6$_5$	616.6	622.0	627.2	631.8	636.7$_5$
89.9	94.2	98.5$_5$	02.9	07.2	11.7	16.6	22.0	27.1$_5$	31.8	36.7$_5$
90.1	94.4	98.8$_5$	03.2	07.4$_5$	11.8	16.6$_5$	22.0	27.1	31.8	36.7$_5$
90.2	94.5	99.1	03.4	07.6	12.0	16.6$_5$	22.0	27.0$_5$	31.8$_5$	36.9
90.3$_5$	94.7	99.4	03.7	07.9	12.2	16.7	22.0	27.0$_5$	31.9	36.9
90.5	94.9	99.6	04.1	08.1	12.5	16.8	22.1	27.1	31.9	36.8$_5$
90.6	95.2	99.8$_5$	04.4	08.4	12.7	17.0	22.4	27.2	32.0	36.8$_5$
90.6$_5$	95.4	600.1	04.6	08.7	12.9	17.2	22.5	27.4	32.0$_5$	36.9
90.7	95.5	00.3	04.8	09.1	13.2	17.5	22.7	27.5	32.1$_5$	37.0
90.7$_5$	95.7	00.5	05.1	09.3$_5$	13.5	17.8	22.9	27.7	32.3	37.0$_5$
90.8	95.8	00.6$_5$	05.3$_5$	09.6	13.9	18.1	23.1	27.8	32.4	37.1
90.8	95.8$_5$	00.7$_5$	05.5	09.8	14.3	18.4	23.4	28.1	32.6	37.2
90.9	95.9	00.8	05.6	10.0	14.5	18.7	23.6	28.4	32.7	37.4
90.9$_5$	95.9$_5$	00.8$_5$	05.7	10.1$_5$	14.7	19.0	23.8	28.6$_5$	33.0	37.5
91.0	96.0	00.9	05.7$_5$	10.2	14.8	19.2	24.1	28.9	33.2$_5$	37.7
91.1	96.0	00.9	05.8	10.3	14.9	19.4	24.4	29.1	33.3	37.9
91.2	96.0$_5$	00.9	05.8$_5$	10.3	15.0	19.7	24.6	29.3	33.4	38.1
91.3	96.1	00.9	05.9	10.4	15.1	19.9	24.8	29.5	33.6	38.4
91.4	96.1$_5$	00.9	05.9	10.4	15.1$_5$	20.0	25.0	29.6$_5$	33.8	38.6
91.5	96.2	00.9	05.9	10.4	15.2	20.2	25.0$_5$	29.7	34.1	38.8
91.6	96.2$_5$	01.0	05.9	10.4	15.2$_5$	20.2$_5$	25.1	29.9	34.2	38.9
91.7	96.3$_5$	01.1	05.9$_5$	10.4	15.2$_5$	20.2$_5$	25.1	29.9	34.4	39.2
91.8	96.5	01.2	06.0	10.5	15.3	20.3	25.2	30.0	34.5	39.2
92.0	96.7	01.3	06.0$_5$	10.5	15.3	20.3	25.2	30.0	34.6	39.2
92.2	96.9	01.4	06.1$_5$	10.6	15.4	20.3	25.3	30.0	34.7	39.3$_5$
92.4	97.1	01.5	06.2	10.7	15.4	20.3	25.4	30.0	34.8	39.4
92.6	97.3	01.7	06.2	10.8	15.5	20.3	25.5	30.1	34.8	39.5
92.8	97.5$_5$	01.9	06.4	10.9$_5$	15.7	20.4	25.4	30.1	34.9	39.6
93.0	97.6$_5$	02.1$_5$	06.6	11.1	15.7$_5$	20.5	25.4	30.2	34.9	39.6
93.1$_5$	97.8	02.3$_5$	06.8	11.2	15.9	20.6$_5$	25.6	30.3	35.0	39.6
93.3	97.9	02.5	07.0	11.3	16.1	20.8	25.7	30.3	35.0	39.5$_5$
93.4	98.1	02.6$_5$	07.2	11.5$_5$	16.3	21.0	25.8	30.4	35.0	39.5$_5$
93.5	98.1	02.7	07.2	11.7	16.5	21.2$_5$	26.0	30.5	35.1	39.5$_5$
93.5	98.1	02.7	07.4	11.9	16.6	21.4	26.2	30.6	35.1$_5$	39.6
93.4	98.1	03.0	07.5	12.1	16.6	21.6	26.3	30.7	35.3	39.6
93.3$_5$	*97.9	03.1$_5$	07.7	12.2	16.8	21.8	26.3	30.8	35.4$_5$	39.6
93.3$_5$	98.0	03.2	07.8	12.2$_5$	16.9$_5$	21.9	26.5	30.9	35.4$_5$	39.6$_5$
93.3	97.9$_5$	03.0	07.6	12.3	17.1	22.0	26.7	31.0	35.4$_5$	39.7
93.2	97.9	02.9	07.6	12.3	17.1	22.0	26.8	31.1	35.6	39.9
93.1	97.8$_5$	02.8$_5$	07.4$_5$	12.3	17.0$_5$	22.1	26.9	31.2$_5$	35.7	40.2
93.0$_5$	97.8	02.8	07.3	12.2	17.0	22.2	27.0	31.4	35.9	40.3
93.0	97.7	02.6	07.2$_5$	12.1	17.0	22.3	27.1	31.6	36.1	40.3
93.0	97.7	02.6	07.2	11.9$_5$	16.9	22.3	27.1	31.6$_5$	36.2	40.4
93.1$_5$	97.7	02.5	07.1	11.9	16.8	22.3	27.2	31.7	36.4	40.5
93.3	97.7	02.5	07.1	11.8	16.8	22.1	27.2	31.7	36.5	40.7$_5$
93.4	97.8	02.6	07.1	11.7	16.7	22.1	27.2	31.7	36.5	40.9
93.5$_5$	97.9	02.6	06.9	11.6	16.7	22.0	27.2	31.7	36.6	40.9
93.7	98.1$_5$	02.6	06.9$_5$	11.6	16.6	22.0	27.2	31.8	36.7	40.9$_5$

Observation III (continued)

7	8	9	10	11	12	13	14	15	16	17
641.0_5	645.7	650.3	655.2	659.9	664.5_5	668.7_5	673.1	677.4_5	681.8_5	686.2
41.2_5	—	50.5	55.4	60.2	64.8	68.9	73.2	77.6	81.8_5	86.2_5
41.3_5	—	50.7	55.6	60.5	64.9_5	69.1	73.3	77.7	81.9_5	86.3_5
41.3_5	46.1	50.9	55.8	60.7	65.2	69.4_5	73.5	77.8_5	82.1	86.5_5
41.4_5	46.2	51.0	56.0	60.9	65.4_5	69.7	73.7	78.0_5	82.3	86.7_5
41.5	46.3	51.2	56.3	61.1	65.6	69.8_5	73.9_5	78.2_5	82.5	86.8_5
41.5	46.3_5	51.4	56.4	61.3	65.8	70.1	74.2	78.5	82.8	87.1
41.5	46.3_5	51.5	56.5_5	61.5	66.0	70.3	74.4_5	78.7_5	83.0	87.3_5
41.5_5	46.4	51.6	56.6_5	61.7	66.1_5	70.5_5	74.6_5	79.0	83.2_5	87.5
41.7	46.5	51.7	56.8	61.8_5	66.3_5	70.8	74.8_5	79.2_5	83.4_5	87.7_5
41.8	46.5	51.8	56.9	61.9_5	66.5_5	70.9_5	75.1	79.5	83.6_5	88.1
41.9	46.6	51.7	57.0	62.0_5	66.7	71.0_5	75.3	79.7	83.8	88.3_5
42.0	46.7	51.8_5	57.1	62.1	66.8	71.3	75.4_5	79.9	84.0	88.5
42.2	46.8	51.9	57.1_5	62.2	66.9	71.4	75.5_5	80.0_5	84.1_5	88.7_5
42.3	46.8	52.0	57.1_5	62.2	67.0	71.4_5	75.6_5	80.1_5	84.3	88.9
42.4	46.9	52.1	57.2_5	62.2	67.0	71.4_5	75.7	80.2_5	84.4_5	89.1
42.5_5	47.0	52.2	57.3	62.3	67.0_5	71.4_5	75.7_5	80.2	84.5_5	89.3
42.7	47.1	52.3	57.3_5	62.4	67.0_5	71.4_5	75.8	80.2_5	84.6	89.4
42.9	47.4	52.4	57.5	62.4_5	67.0_5	71.4_5	75.8	80.3	84.6_5	89.4_5
43.1	47.6	52.5	57.6	62.5_5	67.1	71.4_5	75.8	80.4	84.6_5	89.5
43.3	47.8	52.7	57.7	62.7_5	67.2	71.5	75.8_5	80.4	84.6_5	89.5
43.4_5	47.8_5	52.9_5	57.9_5	62.8_5	67.3	71.6	75.9	80.5	84.6_5	89.5_5
43.6	48.0	53.1	58.0	63.0	67.4	71.6_5	75.9_5	80.5_5	84.7	89.6
43.7_5	48.2_5	53.3	58.3	63.1	67.5	71.7_5	76.0	80.6_5	84.7_5	89.6_5
43.8_5	48.5	53.4	58.4	63.2	67.7	71.8_5	76.1	80.7_5	84.8_5	89.7
43.9	48.4	53.6_5	58.5_5	63.3	67.7	72.0	76.2	80.8_5	84.9	
43.9	48.7	53.7	58.7	63.5	67.8_5	72.1_5	76.3	80.9_5	84.9	
44.1	48.8	53.8	58.8	63.7	68.0	72.3	76.5_5	81.0_5	85.0	
44.2_5	48.8	53.9_5	58.9_5	63.9	68.2	72.5	76.7	81.1_5	85.1	
44.1_5	48.8_5	54.0	59.1	63.9_5	68.3_5	72.7	76.9_5	81.3_5	85.2	
44.1	48.9_5	54.1	59.2	64.1	68.5	72.8_5	77.0_5	81.5	85.4	
44.2	49.0	54.1_5	59.3	64.2	68.6_5	73.0	77.2_5	81.6	85.6	
44.2	49.1	54.2	59.3_5	64.2	68.7_5	73.1	77.3_5	81.8_5	85.8	
44.2	49.1	54.2_5	59.4	64.2	68.8_5	73.1_5	77.5_5	81.9_5	86.0	
44.1	49.1	54.2_5	59.4	64.2	68.9	73.2	77.6	82.0	86.1	
44.2	49.1	54.2_5	59.3_5	64.2	68.9	73.2	77.6	82.1_5	86.2_5	
44.3	49.1	54.2_5	59.3_5	64.3	68.9	73.2	77.6	82.2	86.3_5	
44.3_5	49.0	54.2_5	59.3_5	64.3	68.8	73.2	77.6	82.3	86 4	
44.4	49.0	54.2_5	59.3	64.3	68.7	73.1	77.6	82.3	86.4_5	
44.4	49.0_5	54.2_5	59.3	64.2	68.6_5	73.1	77.6	82.2_5	86.5	
44.5	49.1	54.3	59.3_5	64.2	68.6	73.0_5	77.5_5	82.2	86.5	
44.6	49.3	54.2	59.4	64.2	68.5_5	73.0_5	77.5_5	82.2	86.5	
44.7	49.3	54.3	59.4	64.2	68.5	73.0	77.5	82.1	86.4_5	
44.8_5	49.5	54.3	59.3	64.2_5	68.5	73.0	77.4_5	82.0	86.4	
45.0	49.6	54.4	59.4	64.2_5	68.5	72.9	77.4	81.9_5	86.3_5	
45.2	49.7	54.6	59.5	64.3	68.5	72.9	77.3_5	81.9	86.3	
45.4	49.8	54.8	59.6	64.3_5	68.5_5	72.9_5	77.3_5	81.8_5	86.2_5	
45.6	50.1	55.0	59.7	64.4	68.6_5	73.0	77.4	81.8_5	86.2	

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ON SOME RELATIONS BETWEEN THE HOT SPRING AND RADIOACTIVITY

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ABSTRACT

Some studies were made on the relation between the hot spring and radioactivity. The rapid variation of radon-content with time in the thermal water of hot springs was observed at several places, and a newly devised method for measuring such phenomena is mentioned along with a proposed model hot spring to fit the phenomena. A theoretical treatment is also given on the relation between the hot spring, radioactivity and ground temperature to explain the anomalous distribution of intense ground radioactivity around the site of a hot spring. And, moreover, some considerations are made on the source depth of the radioactive matter contained in the thermal water of hot springs; a conclusion is reached supporting the theory of its shallow origin.

1. Introduction

Since the discovery of radium many studies have been made on the radioactivity of hot springs throughout the world, mainly in relation to chemical constituents. In Japan, a place famous for its most abundant and utilized hot springs, numerous observations have been made mainly on the radon-content in the thermal water of hot springs in various districts. The accurate measurements of the radon-content in hot springs made by Y. Iimori (**1**) with the I. M. Fontactoscope, I. Iwasaki (**2**), K. Kuroda (**3**), and other researchers should be highly appreciated. K. Kimura (**4**) was the first to introduce the G–M counter into the measurement of radioactive deposits in hot springs, and W. Kolb (**5**) recently published a paper on a method of measuring the radioactive properties of hot springs. S. Iwasaki (**6**) has recently examined the connection between hot springs and ground radiation. The writer has been investigating some geophysical phenomena in relation to ground radioactivity since 1949 (**9**, **10**, **11**, **12**, **13**), and in the present article are reported some studies on the relation between hot springs and radioactivity.

2. Variation of radon-content with time in the thermal water of hot springs

The radioactivity of hot springs has been precisely measured by several researchers, reaching a numerical accuracy of expression in the thousandths. And the

difference between the measured values of the radon-content of a particular hot spring in different periods from one to several years' duration has also been examined. But recently I. Iwasaki (**7**) discussed the great fluctuation in the measured value of radon-content which he had observed occurring once in three hours at a particular hot spring. On the other hand, the writer has been also investigating the same phenomena from a different point of view, and its result will be reported in some detail below.

(*a*) *Method of measurement*

The quantity of radium content in liquid has hitherto been usually measured with the I. M. Fontactoscope or the Lauritsen electrometer specially devised by K. Kimura and K. Kuroda (8). But these methods are not considered to be unconditionally suitable for a repeated measurement of short duration, because of the radioactive contamination by radon and thoron which is inevitably induced in such sort of measurements. To avoid this inconvenience the writer devised a special method for measureing rapidly and accurately the radon-content of hot spring thermal water (**10**) and tried to detect the minute fluctuation of hot spring radon-content in the short duration of several minutes.

The procedure of measurement is briefly summarized as follows.

1) To fill the glass bottle with the hot spring thermal water at the spout and seal it hermetically, excluding bubbles.

2) To leave the sampled bottle alone for three hours.

In these three hours the radon in the sampled thermal water reaches a state of equilibrium with its decay products (see Fig. 1). And the above-mentioned three hours is the minimum duration for sufficient saturation.

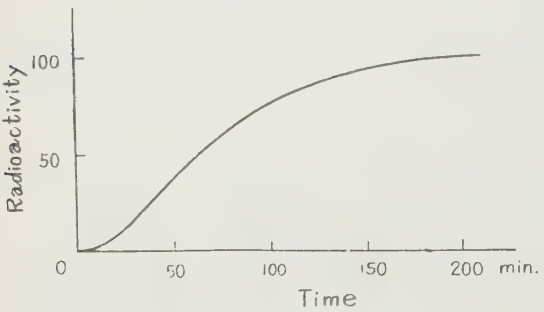

Fig. 1. Saturation curve of radon's decay products measured with β-rays and γ-rays.

It ought to be remarked that the errors resulting from the difference of time of arrival from the subterranean radioactive source are negligibly small because of the long half-life of radon.

An air gap is formed in the upper part of the sampled bottle caused by the difference in thermal expansion coefficient between the water and glass in temperature drop during the three hours' storage, but the radon-content in this air gap was ascertained by the writer (**10**) to be less than 0.1% of the radon-content in the sampled water.

3) To take the water sample after the three hours' storage, and add to it a

certain amount of sulphuric acid and then a certain amount of barium nitrate solution. In this case the 9.5 mg barium nitrate is sufficient for the sample water of 25 cc. Then to filtrate the solution with the combined precipitate of decay products with barium sulphite. The process of this stage needs nearly five minutes and its induced errors are estimated to be less than 5%.

4) To wrap quickly the filtrating paper deposited with the decay products of radon into the stencil paper of copy print in thickness of 1.5 mg/cm^2. This method is considered to be very convenient and efficient for avoiding the injurious effect of radioactive contamination (**11**).

5) To count the radiation number derived from the deposits of radon decay products by quickly winding the stencil paper containing deposits around the G–M counter of the tube type or sticking it to the window of the counter of the end window type.

6) It is to be remarked that sampled water containing thoron, radium, thorium and other elements should be differently handled from the above process, but practically speaking, the existence of these elements does not injure the accuracy of the present method of measuring the short time fluctuation of radon-content in hot spring thermal water, because radium, thorium and decay products of thoron have long half-life compared with that of decay products of radon.

By the above-mentioned method the radon-contents in the thermal water of the hot spring could be accurately and regularly measured once in five minutes. Although there is another method of continuous observation by using the dipping counter, it was proved (**12**) to be inappropriate for the present investigation for detecting short period fluctuations in radon-content because the radon decay products in spring water sticking to the counter tube greatly injure the measured value, and moreover the obtained count by the dipping counter is small compared with the natural count. The count obtained by the present method may rightly be transferred to the intensity of radon-content by using the table previously compiled from the comparison experiment between the count and the mache-number of the solution of a known radon-content by the aid of the I. M. Fontactoscope. The G–M counter used in the present investigation will be described in some detail in § 3. The I–M Fontactoscope used for comparison is the apparatus made by the Scientific Research Institute in Tokyo.

(b) Variation of radon-content with time

The short time radon-content variations in hot spring thermal water were observed at three hot springs, namely, Misasa and Hamamura (both in Tottori Prefecture), and Ikeda (a mineral spring in Shimane Prefecture) by the above-described method. And their observed results are shown in Figs. 2, 3, and 4, whose ordinates and abscissae represent the radon-content in mache and time in hours respectively. It is to be

remarked that the quantity of gushing water differs greatly, those being about 1 litre per minute at Misasa, about 2 litres per minute at Ikeda, and about 10 litres per minute at Hamamura respectively. And, both Misasa and Ikeda are situated at the gorge of a mountain of igneous rock, and on the other hand, Hamamura on an alluvial plain. As clearly seen in the figures, the radon-content in the thermal water of the hot springs or the cold water of the mineral spring varies greatly within the con-

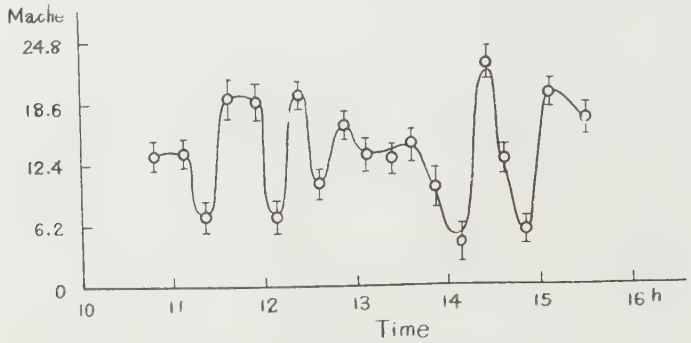

Fig. 2. Variation of radon-content with time at the Misasa hot spring.

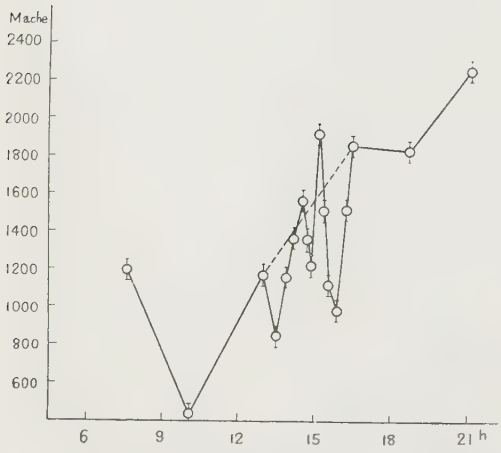

Fig. 3. Variation of radon-content with time at the Ikeda mineral spring.

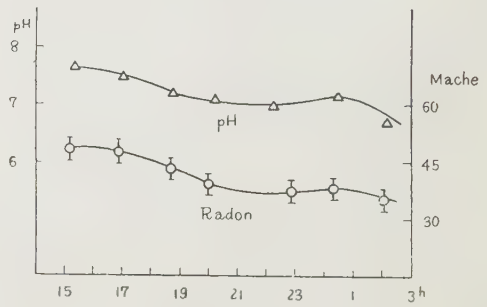

Fig. 4. Variation of radon-content with time at the Hamamura hot spring.

siderably short time of fifteen minutes. It may safely be said that a shorter time variation should certainly exist and could be detected by the above-described method, as the time required in the measuring process is within only five minutes and the successive observation per five minutes or less is possible in the case of a sufficient number of observers. Consequently, the hitherto discussed problems on the radon variation, as deduced from the observed data of every several hours or every several

days should carefully be examined in consideration of the violent short time changes revealed by the present observation.

(*c*) *An explanation for the observed short period time variation of radon-content.*

It is well known that the total intensity of radioactivity in a liquid containing fresh radon gradually increases as the radon decays with time, producing radium A, radium B, etc. to reach the saturation state in nearly three hours after the initial radon-absorption in the liquid as shown in Fig. 1. And the radon itself decays with its half-life of 3.8 days. In the present investigation the comparative measurement of radioactive intensity in the thermal water at the instant of gushing out with that after the time of saturation was tentatively applied for the purpose of explaining the nature and mechanism of time variation of radon-content of the thermal water in a short period. Namely, the age of radon was calculated from the ratio of the radioactive intensity at the gushing instant to that after saturation by applying the saturation curve of radon in Fig. 1. The value of $\frac{2}{3}$, of the ratio, for instance, obtained by comparison of radioactive intensity in the initial and final states would be interpreted to tell that the radon in question is 80 minutes old. It should be remarked here that the hot water in question will certainly contain radons of various ages, and correspondingly the value of age obtained by the above method is considered to show vaguely the mean age of accumulated radons of various origins and birth times. But in consideration of the fact obtained in the practical measurement that the ages of radon thus determined were all within 100 minutes, it is safely and rightly

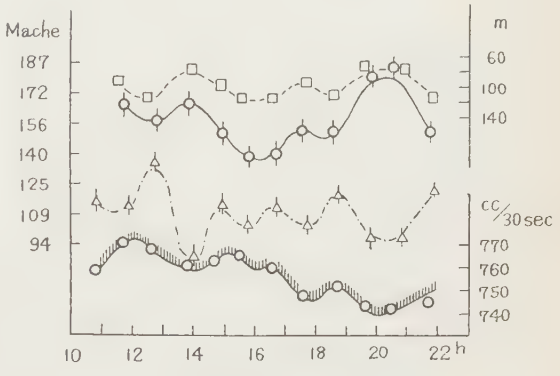

Fig. 5. Misasa hot spring (Spring of the Inn Yasui) July 25, 1951.

- - - - - Age of radon
——— Total radon-content
—•—•— Radioactivity of the thermal water measured as soon as it gushed out
Quantity of the thermal water

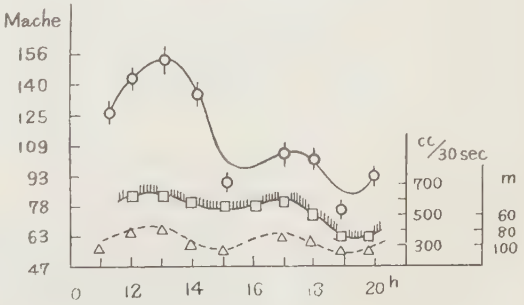

Fig. 6. Misasa hot spring (Spring of the Inn Yasui) July 15, 1951.

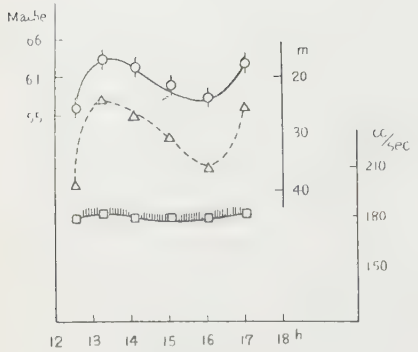

Fig. 7. Hamamura hot spring (Asinoyu of the Inn Tasumoto) July 13, 1951.

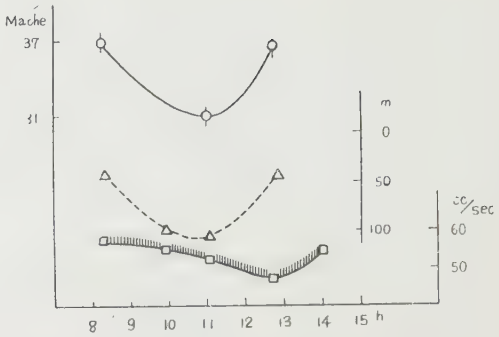

Fig. 8. Sekigane hot spring (Spring of the Hotel Onseiro) July 26, 1951.

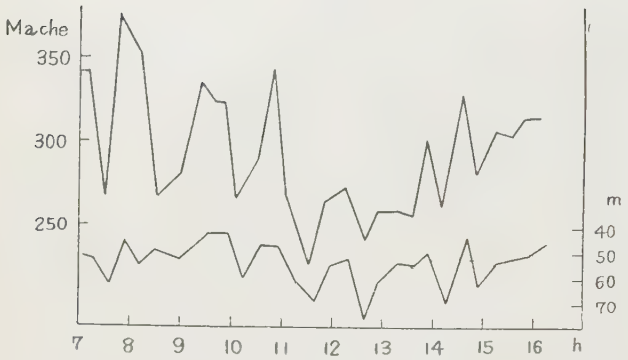

Fig. 9. Ikeda mineral spring. Aug. 15, 1955.

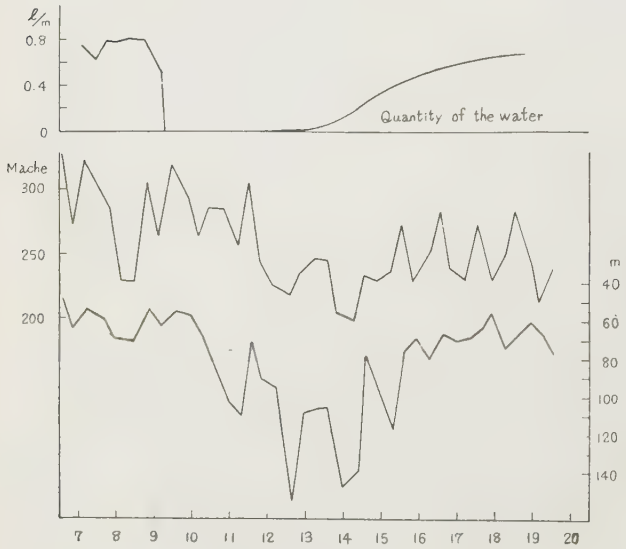

Fig. 10. Ikeda mineral spring. Aug. 16, 1955.

assumed that the radon or the group of radons treated in the present measurement are mainly concerned with those of comparatively younger or fresh radon. In Figs. 5, 6, 7, 8, 9, and 10, are plotted the observed date at Misasa, Hamamura, and Sekigane hot springs (in Tottori Prefecture), and Ikeda mineral spring (in Shimane Prefecture) with regard to the quantity of thermal water at the spout, the radon-content of the thermal water, and the age determined by the above-described procedure. In Fig. 5 the intensity of radio-activity measured soon after the gushing out is also plotted as an example, and in Fig. 10, the observed data at the same spring shown in Fig. 9 are given

in the case of an artificial stoppage of gushing mineral water for several hours.

From all the above figures, the following relationships between the gushing thermal water, the radon-content, the age of radon in the above described sense and amplitude of time variation of radon content are clearly observed.

(i). The younger the radon is, the stronger its radioactivity is, at the one and same spring.

(ii). The smaller the quantity of gushing thermal water, the more severe the variation of radon-content with time in case of comparing the two springs in the different districts.

(iii). The time variation of radon-content is intimately related to the quantity of gushing thermal water at some springs, while there is no connection at other springs.

As above stated, the radon concerned in the present measurement, especially for the problem of its time variation, is reasonably considered to be comparatively young or fresh, and consequently the time variation of age of radon is interpreted as mainly relating to the difference of path through the underground vein of the thermal water containing fresh radon. Roughly speaking, we may explain these phenomena by saying that the shallower the underground radon source, the stronger radon intensity in the hot spring thermal water measured at the ground surface. Speaking in more detail, however, there may be many small cavities in the underground and, in the case of a radioactive spring, a considerable amount of radium and other radioactive elements is considered to be precipitated in these cavities. Under these circumstances the intensity of radon-content in the thermal water of a radioactive spring will be greatly influenced by the position or simply the depth of the concerned small cavity containing the strong radioactive liquid which largely contributed the fresh radon to the neutral thermal water. It is generally considered that the thermal water activated by the radioactive liquid in the shallow cavity is less disturbed and weakened by the mingling of the underground water and other disturbances compared with those in case

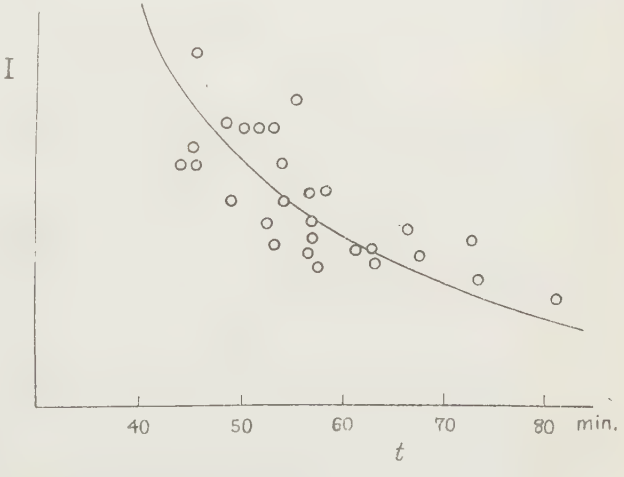

Fig. 11. Relation between the radon-content and their ages.

of deep cavities. Under these assumptions the above-obtained relation between the radon-content in the thermal water and the age or freshness of radon as mentioned in (i), will reasonably be explained. From Fig. 11, which represents graphically the relation observed between the radon-contents and their age of the data plotted in Fig. 9, the following empirical formula is obtained, denoting the radon-content in the thermal water and the age of radon as I and t respectively:

$$I = \frac{a}{t^2} + b,$$

where a and b are numerical constants proper to each spring. Namely, the above formula expresses that the intensity of radon measured in the thermal water of any one spring is inversely proportional to the square of its age. After the assumed model or way of radioactivation of the originally neutral thermal water by the strong radioactive liquid in the small underground cavities, the mode of rapid variation of radon-content in the thermal water will also approximately be explained. The phenomenon of rapid radon-content variation is probably caused by difference of contributed or enriched amounts of fresh radon to the neutral thermal water by the underground small cavities at various depths and having strong radioactive liquids of variable intensity. As the underground radioactive cavities in question are considered to correspond to the micro-pores and micro-fissures in case of hot springs with rocky foundations, and the capacity for containing a strongly radioactive liquid in the cavity is estimated to be very small, the radon-content in the thermal water measured at the ground surface will correspondingly show a rapid and violent fluctuation, especially in case of hot spring with small quantity of gushing water, according to the rapid consumption of strong radioactive liquid in the small cavity, and the rapid alternation of the cavity as the furnisher of fresh radon to the neutral thermal water. Of the hot spring with a sand or clay foundation the time variation of radon-content observed is comparatively small and this is reasonably explained by the circumstances that there are probably no distinct underground cavities to show rapid consumption and alternation in furnishing the fresh radon to the neutral water, and moreover, the effect of enrichment of fresh radon to the neutral water, if any exist, may greatly be weakened by the diluting effect of the great quantity of underground water abundant in the sandy layer. In concluding this section, the rapid and violent fluctuation of radon-content in hot spring thermal water was observed at the same hot springs by the improved method of rapid measurement of the thermal water radon-content. And a model radioactive hot spring was tentatively proposed as an explanation of this phenomenon of radon-content time variation. The existence and effect of small underground cavities containing strong radioactive liquid (cf. **33**) are considered as an approximate explanation of some of the relations obtained in the observation of rapid fluctuation of radon-content in the thermal water.

3. Relation between ground radioactivity and hot springs

It is generally known that the quantity of radon-content in the ground surface or in the superficial layer of the ground is largely influenced by the existence of the fault or underground vein, and the problems have been fully discussed by Z. Hatsuda (**14**), T. Ochiai (**15**), S. Sano (**16**), and other researchers. On the other hand, a correlation between the hot spring and the quantity of radon-content at the ground surface can probably be postulated, in consideration of the circumstances that a large number of hot spring are assumed to originate from underground fissures and veins. But the problem should not be so simply treated, because there are actually various types of hot spring. For example, in the mode of thermal water gushing, there are many types of natural gushing and pumping up, surface origin and by deep boring. And in radioactive intensity, there are strongly radioactive springs and non-radioactive springs. Accordingly, for the purpose of obtaining any relation between ground radiation and hot springs, the precise measurement of ground radioactivity should be applied to as many and various types of hot springs as possible. In our country S. Iwasaki (**6**) has recently reported on this problem for one example of hot spring. In the present investigation, the relations between ground radioactivity and hot spring origin were studied by the precise observation of the ground radiation of nearly twenty hot springs of various types and under different circumstances. It is here to be remarked that the anomalous distribution of ground radioactivity measured and discussed in the present investigation is considered to be mainly concerned with the decay products of radon, because the thoron and its decay products, if existing, cannot become the object of observation for detecting the anomaly of ground radiation in somewhat deep underground layer. We shall refer to this point in some detail in a later section.

(*a*) *Method of measurement*

Z. Hatsuda (**14**) made various applications of the "radon-in-soil-air method", "ground-hole-ionization chamber method", "photo-plate method", and "G–M counter method" in his study on radioactivity of the ground near the faults and other places. And T. Ochiai (**15**) and S. Sano (**17**) used the method with the G–M counter in the research of ground radioactivity. In the present investigation of ground activity near the hot springs the observation was made by the G–M counter method and the soil-air method with the Lauritsen electrometer as described in the following:

The method with the G–M counter—In the previous observation a G–M counter tube made by the writer containing alcohol and air was used with a wall-thickness of 0.2 mm of duralumin, a diameter of 35 mm, and a length of 80 mm, as shown in Fig. 12, the natural count being in the range of 50 to 60 c. p. m. Afterwards the

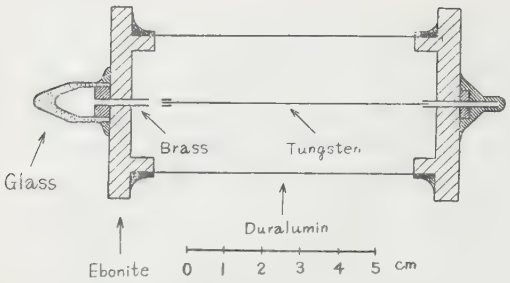

Fig. 12. The structure of the used G M counter.

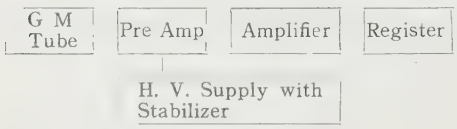

Fig. 13. Circuit of the G-M counter.

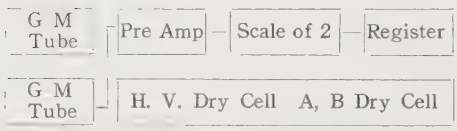

Fig. 14. Circuit of the G-M counter (2s-pl).

G–M counter of end-window type made by the Scientific Research Institute in Tokyo was mainly used in the research observation. This apparatus has a mica-window of 3 mg/cm^2 and shows 30 to 40 c. p. m. as the natural count, and in practice two sets of these counter tubes were used in parallel connection.

In Fig. 13 the connection diagram of the portable counter made by the writer above described is represented. In this case the observation was unprofitably restricted with regard to the place of measurement because of the necessity of applying the source of alternating electric current to the apparatus. In Fig. 14 the connection diagram of the recorder of the end-window counters made by the Scientific Research Institute in Tokyo above described and named (2s–pl), which was somewhat reconstructed for the present investigation, is represented, in which case the dry cells are conveniently used and the parallel sets of two couters brings good results in such a measurement of small counts as in our present observation. The maximum number of accurate counting was estimated to be within 280 c. p. m. in case of the apparatus in Fig. 13, and 800 c. p. m. in case of Fig. 14, but as the present measurement requires a count range of from 60 to 160 c. p. m., both apparatuses would be highly reliable in the point of counting. The duration of counting at one point was in the range of 5 to 10 minutes which was suitably adopted according to the number of counting at the place. And in these cases the errors caused by the statistical fluctuation in a finite duration of counting were estimated to be nearly within 3% of the counts. And the counter tube was efficiently placed, in the practical measurement, at the height of 15 cm above the surface of ground, because measurement at a position lower than 15 cm is considered to be too much sensibly disturbed by the very small and local irregularity of the ground, and the measurement at a higher position than 15 cm is also inconvenient for the measurement of weak β-rays and greatly disturbed in the point of accuracy by γ-rays radiating from the other ground. This was ascertained by some trial measure-

ments by the counter at different heights, and an example is represented in Fig. 15. In the practical measurement, the facts were observed that the counts measured at the position on a road higher than the surroundings or on the heapy ground are apparently of small value owing to the escape of radon from the lateral side of the road on the ground. On the contrary, the counts measured at the lateral side of heapy ground or the hill, and on the fissure of a dry rice-field, were in 10 to 20% of the ordinary value largely observed. Next, in the following Figs. 17~31, the values of counts at the respective points in the same area of hot spring are represented in the figure of ratio of the very counts to the minimum counts observed in that area. This method of representation was suitably adopted for the elimination of errors caused by the secular change in instrumental ability of the counter, the local character of ground radioactivity of a respective small area, and other inevitable disturbances.

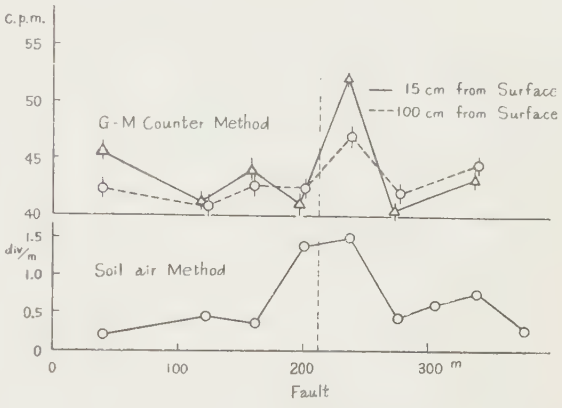

Fig. 15. Survey at the Fault Yoshioka.

The soil-air method with the Lauritsen electrometer—This method is an indirect one in the measurement of soil-air, but convenient in the point of operation. To dig a hole of 10 cm-diameter and 100 cm- or 50 cm-depth in the ground, and, after 3 hours' covering on the hole, suspend a metal plate of $4 \times 6.5\,cm^2$ in the hole, and give the plate a negative charge of 500 volts during 30 minutes. And, next, insert this metal plate into the sample-shelf of the Lauritsen electrometer, and measure the radioactivity of decay products of radon stuck on the metal plate through the aluminum foil window of the electrometer. The Lauritsen electrometer used in the measurement was one made by the Scientific Research Institute in Tokyo and its voltage-sensitivity was estimated as 1.48 div./volt, or 2.30 div./min. for the γ-ray radiated from a radium source of 1 mc at a distance of 1 m from the electrometer. In Fig. 15 an example of concurrent measurement made in the autumn of 1953 is shown with the G–M counter and the soil-air method, the observation side being near the active fault 1 km northward from the Yoshioka hot spring in Tottori Prefecture. But, generally speaking, the soil-air method with the electrometer was not so frequently used as the G–M counter method, because of large errors caused by the local irregularity of the dug hole, and of the heavy labour and time spent in digging the hole and in the processes of measurement.

(b) Daily variation of ground radioactivity

For the purpose of treating the local character of ground activity of any place by the method of radioactive measurement at many points as possible in the area concerned, the effect on the time variation of ground activity by meteorological changes or other like disturbances should fully be examined at each place before the commencement of observation. An example of measurements disturbed by rainfall at the same points, and an example of measurements with daily variations at same points in the area of the Ikeda mineral spring in Shimane Prefecture, are shown in Figs. 16-(a) and 16-(b), which inform us of the large influence of rainfall upon the measured value of ground radioactivity and also, on calm days, the ground-activity remains nearly constant from 9 am to 8 pm. From these observed facts and other experiments, the measurement for the present investigation should preferably be made

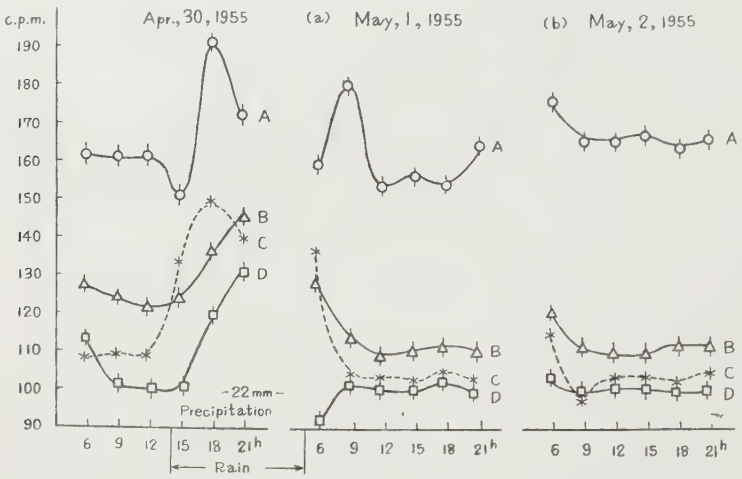

Fig. 16. Daily variation measured at the places of A, B, C, and D near the Ikeda mineral spring.

during the day time between 9 am and 8 pm on calm days, avoiding the rainy days and their following days. And, in case of measurements extending over several days, measurements should repeatedly be made at some reference points during the several days, in consideration of disturbances due to changes in barometric pressure and other causes. As seen in Fig. 16-(b) the ground activity in the early morning shows a large value compared with that in the daytime, and this is resonably explained by the process of release of radon contained in the dew which absorbed radon during the night and evaporated during the early morning, and furthermore by the increase of radon decay products due to the blocking of the capillary openings of the ground surface by the dew.

The increase of ground radioactivity in the case of rainfall will similarly be explained as in the above-described process. The decrease of ground radioactivity at 21h in Fig. 16-(a) which was once largely increased by rainfall of 14h-commencement is explained by the effect of blocking the capillary openings in the deeper layer of the soil by the penetrating water of the rain. From these and as seen in Fig. 16-(a), it is to be strongly remarked that a part of ground radioactivity measured by G-M counter on the ground surface is interpreted as caused by decay products of radon which ascended through the capillary openings in the soil to the ground surface from its original source in somewhat deep underground.

(c) Radioactive measurement at some places with no hot springs

The radioactive measurements at some ordinary places with no hot springs or active faults have been made for comparison in studying the anomalous distribution of ground radioactivity at places near hot springs. Among ten places of ordinary nature recently measured the following two examples will be mentioned as representative: An example shown in Fig. 17 is the spatial distribution of the numerical data of ground radioactivity measured with the G-M counter at the ground surface at Tottori University on an alluvial plain where no fault or other geological irregularity is found. As shown in the figure, it is easily understood that the maximum value of count ratio does not exceed 1.1 at such a place as there is no geological irregularity or anomalous distribution of radioactive substances. In Fig. 18, another example is shown of the measurement at Yudokoro-cho in Tottori City. The

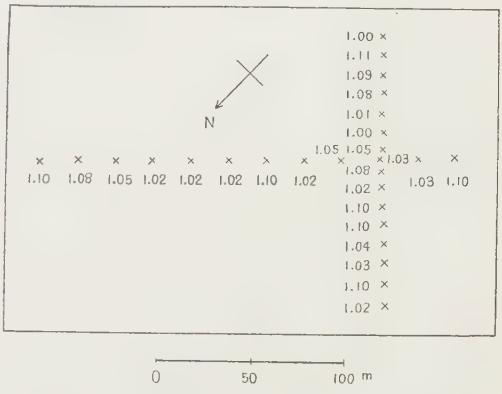

Fig. 17. Survey at the play-ground in the Tottori University.

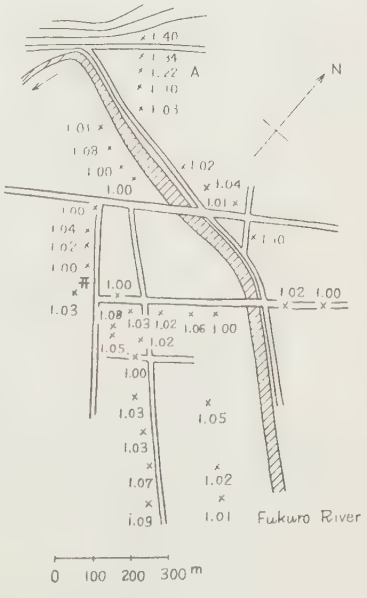

Fig. 18. Survey at Yudokoro-cho in Tottori City.

ground is the alluvial plain of 50 m-thickness, and there is the rock mountain at the north area point A. The count-ratio is specially large in the area A owing to the effect of mountain, but the other major area shows no particularly large values in ground radioactivity. All measurements were made by the G–M counter of the end-window type with two tubes in parallel connection and worked by portable dry electric cells. It is to be remarked that the count-ratio measured at the eight other ordinary places having no geological irregularity show no abnormal value and their differences are all within the statistical fluctuation of counting.

(*d*) *Radioactive measurement at places near hot springs*

The radioactive measurements have been made at many places near hot springs of various kinds for investigating the relationship between hot springs and ground radioactivity. In practical observation, the measuring points were impartially adopted as the circumstances permitted in the area concerned, but carefully selected to avoid the disturbances caused by the influences of hot spring drainage-ditches or the neighbouring houses, structures, pavements, etc. The measurements described in the following comprise thirteen readings which were made at the hot spring areas of Misasa, Hamamura, Togo, Asozu, Kaike, Yosioka, Iwai, Yutani (all in Tottori Prefecture), Kinosaki, Yumura (in Hyogo Prefecture), Akune (in Kagoshima Prefecture), Yunoura (in Kumamamoto Prefecture) and Beppu (in Oita Prefecture). In the following the materials of observation at each place will be represented in some detail.

1. *Misasa hot spring*

The numerical results are shown in Fig. 19. The bed rock of this area is porphyritic granite and it is intruded in spots by the fine grained granite from where

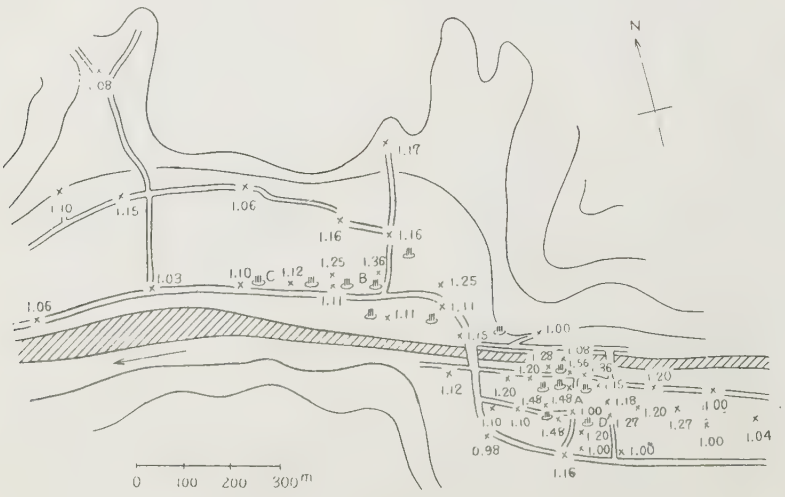

Fig. 19. Survey at the Misasa hot spring.

the hot water springs out. The hot water gushes out naturally at the small area surrounding the two points A and B in the figure, and it needs nearly 30 meters' boring at the area surrounding two points of C and D, there being no hot spring excepting these four points and their surrounding small area. On the radioactivity in the thermal water of hot springs in this area, the contents are all radon except one point in the B-area which contains thoron, and their values are about 100～200 mache in the B- and C- area, and less than 100 mache in the A- and D- area. The temperature of the thermal water is measured as in the range of 40°～80°C in this area.

2. *Togo hot spring*

The numerical results are shown in Fig. 20. This hot spring is situated close to the southern part of the small lake of Togo, and its bed rock is granite which is covered by a clay bed and gravels. The thermal water gushes out by itself through

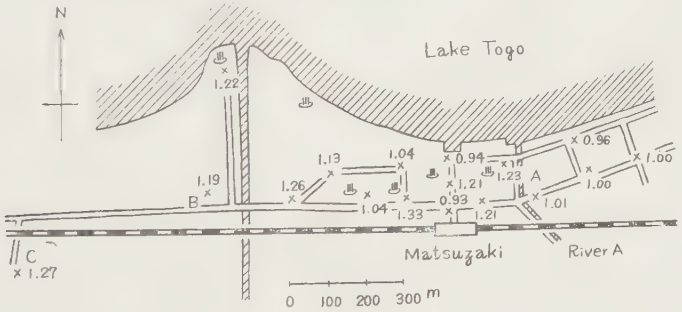

Fig. 20. Survey at the Togo hot sprng.

a boring-tube of about 30 meters. The east side of River A is said to have no hot spring source which fact is well concordant with the small value of the radioactive counts in this area. The large values of counts at the place near two points of B and C may probably be interpreted by the existence of either unexposed faults or a latent hot spring, but its truth should be assertained by future investigation. The value of the radon-content in the thermal water of this area is about 2～18 mache and its temperature is in the range of 42°～54°C.

3. *Asozu hot spring*

The numerical results are shown in Fig. 21. The hot spring is also situated close to the northern part of Lake Togo, and the ground condition is considered to be the same as that of Togo hot spring. The thermal water gushes out by itself through the

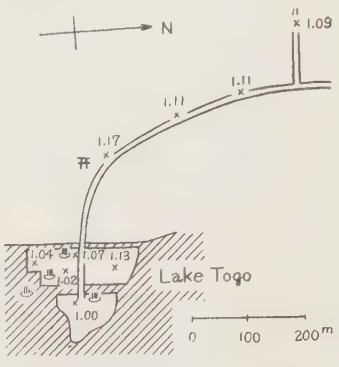

Fig. 21. Survey at the Asozu hot spring.

boring tube. In this case the radioactive counts measured do not show any large values at the point of the hot spring, and this example is the only exceptional case in the present series of observations which does not show any large value in the hot spring area.

4. *Hamamura hot spring and Kachimi hot spring*

The numerical results are shown in Fig. 22. The northern part of the area in the figure is named Hamamura and the southern part is Kachimi, both being considered to be parts of the same hot spring system. In the figure the A–hill is formed by strongly radioactive granite and this rock formation is presumed to extend to the underground in the area of B. The ground surface is an alluvium area with the alternating strata of clay beds and gravels. The temperature of the thermal water at Kachimi is generally higher than that at Hamamura, and the depths of boring are about 10 m and 30 m near the point C in the area of Kachimi and Hamamura respectively. The values of radon-content are measured as 40～60 mache in the small area of D, but in the other area nearly 3～10 mache. The temperature of thermal water is in the range of 40°～70°C, but in the small area of E it is particularly less than 30°C.

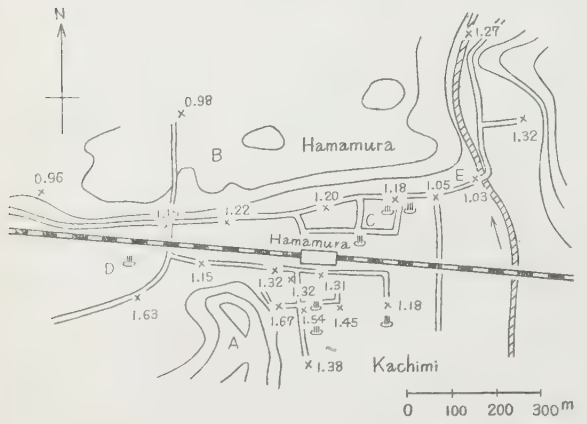

Fig. 22. Survey at the Hamamura and Kachimi hot springs.

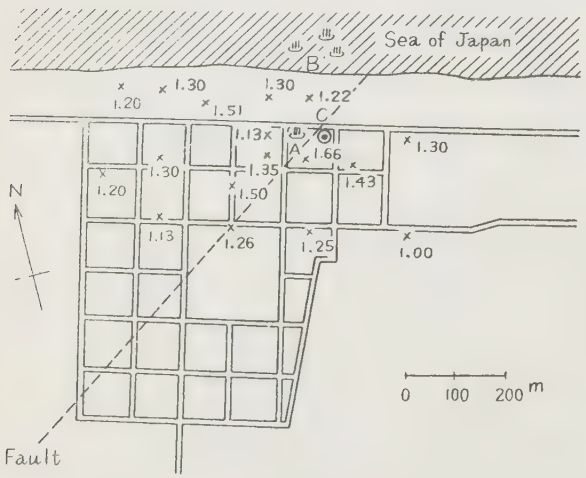

Fig. 23. Survey at the Kaike hot spring.

5. *Kaike hot spring*

The numerical results are shown in Fig. 23. The Kaike hot spring is situated close to the shore of the Japan Sea, and some old hot springs were, as seen in the figure, submerged in the sea water by the severe beach erosion,

and only the hot springs at the two points of A and B are available at present. The depth of boring is about 100 m, and the temperature of the thermal water is in the range of 80°～90°C, the radon-contents being very small in 0～0.5 mache. The bed rock is andesite, being covered by an alternating strata of gravels, sand stone and clay bed with a total thickness of nearly 100 m. In this area the hot water obtained at the spring sources is delivered to each hotel, and, after use, is released without control onto the neighbourihg sandy ground and allowed to be absorbed by the ground, owing to the incomplete sewerage equipment in hotels. It is to be mentioned here that the point C in the figure was recently bored shortly after our measurement and succeeded in obtaining a new hot spring originating from the vein at a depth of 180 m under the ground surface. This point had already been estimated as a promising site for a hot spring both by the analysis of an unexpected fault by the measurement of ground radioactivity with the soil-air method and by the large value of radioactive counts with a G–M counter.

6. *Yoshioka hot spring*

The numerical results are shown in Fig. 24. The rock formation is the mixture of coarse- and fine-grained biotite granite with andesite in spots, covered by the thin layer of surface soil. The hot springs in the present use in this area are all naturally gushing out over the ground surface, and their temperatures and radon-contents run between 40°～50°C and 1～3 mache respectively. As seen in the figure, there are many points having no hot spring at the present time, in spite of the comparatively large value of ground radioactivity measured. The problem is now in question as to whether these large values of ground radioactivity are caused either by some crack and fissure in the bed rock, or by the latent source of hot spring, and is awaiting future investigation. Recently, the boring work of 30 m at the point A has failed to obtain a new hot spring, and this point coincides unfortunately with the area of small value of ground radioactivity.

Fig. 24. Survey at the Yoshioka hot spring.

7. *Iwai hot spring*

The numerical results are shown in Fig. 25. The geological formation at Iwai hot spring is a complex of tertiary tuff, shale and sandstone with some liparite in the western area. The temperature and radon-content in the thermal water is in the range of 35°～50°C and about 1 mache respectively. In the area north of the Gamo

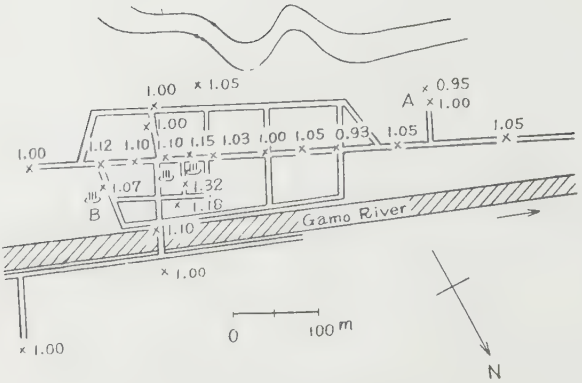

Fig. 25. Survey at the Iwai hot spring.

River and around the point A it was ascertained by boring that there was no hot spring source which finding well corresponds with the small value of ground radioactivity observed in these areas. A hot spring was obtained at the point B at a depth of 30 m.

8. *Yudani hot spring*

The numerical results are shown in Fig. 26. The Yudani hot spring is in an area of tertiary tuff, there being a natural gushing and a temperature of about 30°C. The thermal water contains NaCl and its radon-contents are 4~7 mache. The area

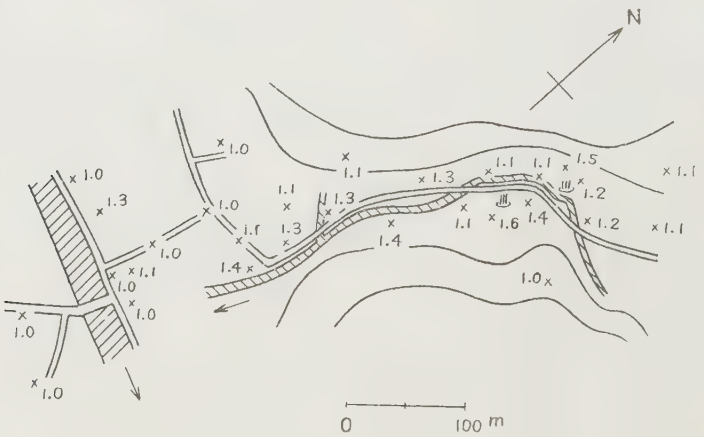

Fig. 26. Survey at the Yudani hot spring.

around the hot springs shows a large value of ground radioactivity which may presumably be explained either by an unexpected fault or by the influence of high underground temperature, but this supposition is being questioned at the present time.

9. *Kinosaki hot spring*

The numerical results are shown in Fig. 27. The geological formation in the area of Kinosaki hot spring is layered by a granitic bed, an intermediate layer of tertiary tuff with liparite-dike, and a surface alluvium layer. The temperature and radon-content in the thermal water are in the range of 50°～59°C and 1～4 mache respectively.

Fig. 27. Survay at the Kinosaki hot spring.

10. *Yumura hot spring*

The numerical results are shown in Fig. 28. The Yumura hot spring is situated on a layer of tertiary tuff, and in the area of the river-side. The hot water of 94°～97°C gushes out naturally, but the radon-content in the thermal water in the area of Yumura is generally small, being less than 0.5 mache.

11. *Akune hot spring*

The numerical results are shown in Fig. 29. The Akune hot spring is situated close to the sea shore on an alluvium plain, and the spring contains salts, the neighbouring salt-farm being

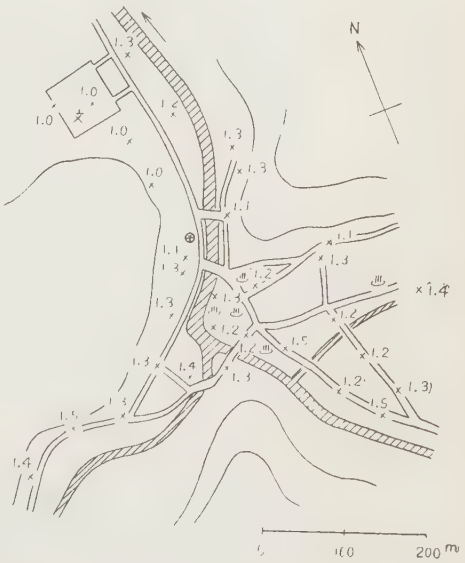

Fig. 28. Survey at the Yumura hot spring.

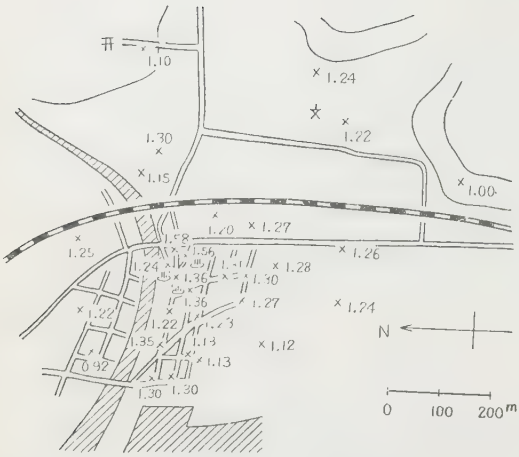

Fig. 29. Survey at the Akune hot spring.

managed by collecting the saline constituent in the underground water. The springs are all made by the 25～30 m's boring, the temperature and radon contents of their thermal water being about 43°C and 5～12 mache (18) respectively.

12. *Yunoura hot spring*

The numerical results are shown Fig. 30. In this area the hot water is considered to be gushing out through vein of andesite. In the period of measurement the point A was in the operation of boring, but it is said that the operation of boring was recently suspended for the reason that the temperature of water obtained by the 50-m's boring was less than 25°C. And this fact is fairly concordant with the small value of ground radioactivity, at this very point. On the other hand, the hot springs at a distance of 20 m west from the point A are

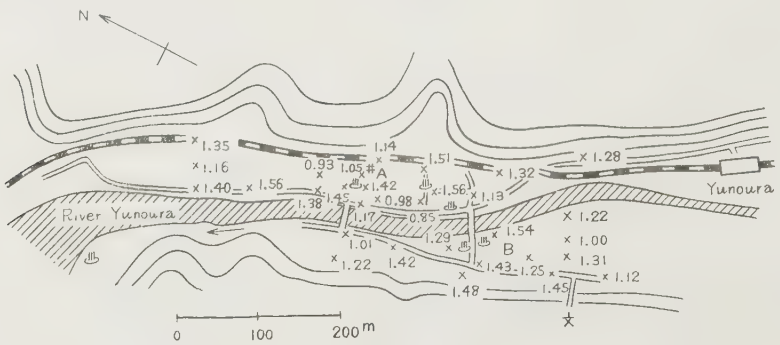

Fig. 30. Survey at the Yunoura hot spring.

gushing out thermal water of 43°C originating from a source 30 m below the ground surface, and the ground radioactivity at this point shows, as seen in the figure, a very large value compared with its surrounding area. And also with regard to the point B the above-described relation is fairly assertained.

13. *Beppu hot spring*

The numerical results are shown in Fig. 31. In the area of the Beppu spring various investigations on the nature of hot spring have been made by the research members of the Beppu Balneological Laboratory attached to Kyoto University. In the

figure the distribution of hot spring temperatures measured by T. Nomitsu and K. Yamashita (**19**), and K. Yamashita (**20**) is represented for comparison with the distribution of ground radioactivity measurements with the G–M counter, and the existence of an intimate correlation between them was also ascertained in this case. In the figure all the roads and hot springs in Beppu City are not represented to avoid complication, but generaly speaking, there are found many hot springs in the area of high ground temperature. Namely, the small area between Beppu Port and the west from the Beppu Railroad Station is the area most crowded with hot springs,

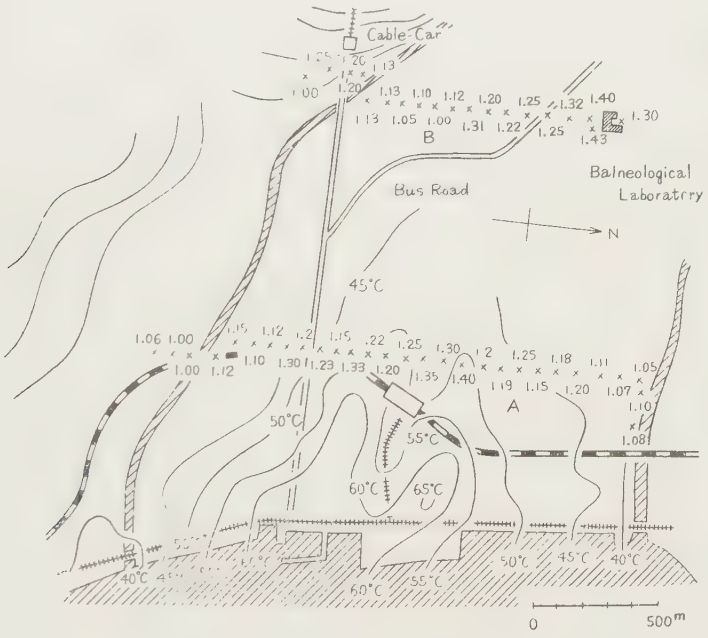

Fig. 31. Survey at the Beppu hot spring.

and, in addition, there are areas of hot springs at the far distant part south from the Port, at the small part around the cable-car station in the west end of the City. It is to be remarked that the small area around the Balneological Laboratory shows a large value of ground radioactivity, and it is said that the hot spring is not yet obtained because of low level of ground water inspite of the existence of hot spring source underneath the ground around the Laboratory. In the area of Beppu the hot spring thermal water contains few radon of 0~1 mache, and the bed rock under the area is basalt.

(*e*) *Concluding remarks on the results of measurement*

The above-described measurements of ground radioactivity in some areas of hot spring were carefully made during the daytime between 9 am and 5 pm avoiding the

rainy day and its following day. In a general review of the present measurements the following characteristics will be noticed:

1. In a hot spring area the ground radioactivity at the very site of the spring is much larger compared with that at a remote place.
2. The above-described relation between hot spring and ground radioactivity holds well in all cases of either radioactive or non-radioactive hot springs.
3. The hot water of non-radioactive springs is considered to be generally derived from the underground layer of bed rock, and really an area far remote from the spring site shows sometimes a comparatively large value of ground radioactivity in such cases as where the hot water concerned is supposed to flow along the common underground bed rock of two sites of the spring and the remote ordinary place. From these facts it is reasonably concluded that the ground radioactivity is greatly influenced by the underground temperature in addition to the existence of faults and veins or fissures.
4. Along the supposed stream of underground water, the ground radioactivity in the area of a hot spring on the upper course of the underground water-stream is generally stronger than that in the area on the lower course.
5. In the course of or after the measurement of ground activity in the areas of hot springs, there were reported the four examples (Kaike, Katsutani, Kinosaki, Misasa) of success in obtaining new hot springs bored at the areas of strong ground radioactivity, and other four examples (Yoshioka, Misasa, Yunoura, Tottori) of failure in obtaining new spring bored at the areas of weak ground radioactivity.

4. Theoretical consideration on the results of measurement

(*a*) *On the density of soil-radon*

In the preceding section the fact was found that the ground radioactivity of the area near a hot spring is much stronger compared with that of a more remote area, and this relation holds for any radioactive or non-radioactive hot spring. For the purpose of explaining this phenomenon, the underground distribution of radon-density, taking the underground temperature into consideration, will theoretically be treated below.

It was formerly ascertained by E. Buckingham (**21**), J. Koenigsberger (**22**), and Z. Hatsuda (**14**) that the motion of gas molecules in the underground layer may be treated in approximately the same manner as those in the free air under the circumstance that the air gaps between the soil particles are larger than the mean free path of gas molecule. And, moreover, it was proved by E. McDermott (**23**) that the underground gas even in shale can reach the ground surface through a small underground fissure, and G. Aeckerlein (**24**) formerly reported that the radon does the

main upward diffusion in the underground. From these results of study the density of soil-radon will theoretically be treated as follows.

It is assumed that the underground soil or rock contains uniform amounts of radium, and that radon is derived from these radioactive sources. The amount of radon contained in a unit volume of any underground point is the sum of the newly derived radon at the very point and the diffused radon from the underlying layer, and its certain amount is dissolved in the underground water, its amount being determined according to the partition influenced by the temperature at that point. The dissolved radon decays at that point and does not ascend because of its small diffusion coefficient, and consequently the amount of radon decayed is supplied from the radon source in gas. And it is also assumed that the porosity of the underground layer and the water-content among the sand particles are everywhere uniform. Denoting the amount of radon radiated from a unit volume of soil and contained in the underground gap, the water-content in the underground gap, the density of radon in the underground gap, and the density of radon in the underground water, by Q, k, ρ, and ρ' respectively, the ratio of ρ' to ρ becomes

$$\frac{\rho'}{\rho}=f(T),$$

where $f(T)$ is a certain function of partition rate with the underground temperature T as a parameter. And the above relation will be represented graphically as in Fig. 32. In a first approximation the relation between ρ'/ρ and underground temperature is roughly formulated in the range of temperature of 20°~50°C as

$$f(T)\sim g-lx\,,$$

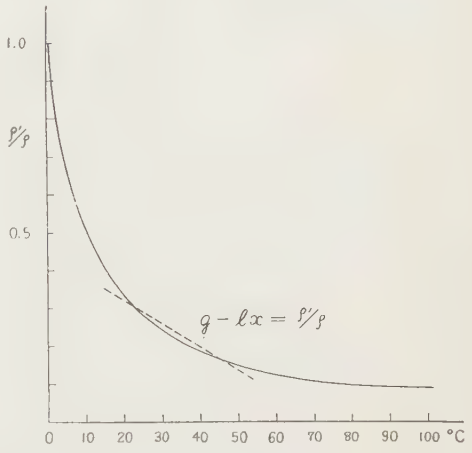

Fig. 32. Relation between ρ'/ρ and the ground temperature.

where g, l are some constants proper to the area concerned, and x is the depth of the point in question from the ground surface, the temperature being assumed to be linearly proportional to x in this case. We thus have

$$\rho'=(g-lx)\,\rho\,,$$

and consequently the radon-content dissolved in the underground water is calculated as

$$k(g-lx)\,\rho\,. \qquad (1)$$

In case when the underground porosity (n) is taken into consideration, the above radon-content in the water becomes

$$nk(g-lx)\rho\,. \tag{2}$$

Under the circumstances that the amount of decay in the radon-content expressed by (2) is supplied from the Q in the gap and moreover the radon-content dissolved in the water makes little contribution to the radon of upward diffusion according to its small diffusion coefficient, the amount of radon radiated in the gap will be calculated as

$$Q-\lambda nk\rho(g-lx)\,, \tag{3}$$

where λ ($=2.1\times10^{-6}$ sec^{-1}) is the decay constant of radon. Considering these relations and denoting the diffusion coefficient by D, the general equation of diffusion of underground radon becomes

$$n\frac{\partial\rho}{\partial t}=nD\frac{\partial^2\rho}{\partial x^2}-n\lambda\rho(1-k)+Q-\lambda nk(g-lx)\rho\,.$$

In the present treatment on the stationary state of underground radon, $\partial\rho/\partial t$ can be put equal to zero and consequently the differential equation for the density distribution of underground radon may be expressed as follows:

$$\frac{\partial^2\rho}{\partial x^2}-\frac{\lambda\rho}{D}(1-k+kg-klx)=-\frac{Q}{Dn}\,. \tag{4}$$

From the kinetic theory of gas, D is expressed by the formula, $D=T^{\frac{1}{2}}/\gamma\times$const. And the ratio of the diffusion coefficient at 50°C unit with that at 20°C is expressed as $D_{50°}/D_{20°}=\sqrt{323}\,\gamma_{20°}/\sqrt{293}\,\gamma_{50°}=1.05\times\gamma_{20°}/\gamma_{50°}$. In a first approximation, by putting $\gamma_{20°}\cong\gamma_{50°}$, $D_{50°}/D_{20°}\cong1$, and consequently, in a rough calculation, D is treated as a constant in any state of temperature. From the table in G. V. Heavesy and F. A. Paneth's Manual (**25**), D is estimated nearly as 0.07 cm^2 sec^{-1} at the condition of 760 mm Hg at room temperature, and in this case the eddy diffusion may reasonably be neglected. Then, denoting the quantities in Eq. (4) abridgedly as $kl=a$, $\lambda(kg+1-k)/D=m^2$, $Q/Dn=\sigma$, Eq. (4) becomes

$$\frac{\partial^2\rho}{\partial x^2}-\left(m^2-\frac{\lambda a}{D}x\right)\rho=-\sigma\,. \tag{5}$$

In a practical case, taking k as 0.3 and calculating $l_{50°}$ by dividing the difference of ρ'/ρ between the value at the underground temperature of 50°C at 40 m-depth and the surface ground temperature of 18°C, with x ($=40$ m), in Fig. 32, we get

$$l_{50°}=0.2/4000=5\times10^{-5}\,.$$

Similarly, by calculating $l_{20°}$ in the case of an underground temperature of 20°C at 40 m-depth and a surface ground temperature of 18°C, we get

$$l_{20°}=0.03/4000=7\times10^{-6}\,.$$

From the above calculation, as $kl_{50°}=a_{50°}=1.5\times10^{-5}$, $\lambda a_{50°}/D=4.5\times10^{-10}\ll1$ and the quantity of $\lambda a/D$ with regard to 50°C and 20°C is estimated as more and more smaller values, the term of $\left(\frac{\lambda a}{D}x\right)$ is small compared with the term of m^2 in Eq. (5). Consequently, Eq. (5) is approximately treated as in the following expression in the method of solution by perturbation:

$$\frac{\partial^2\rho}{\partial x^2}-m^2\rho=-\sigma\,. \tag{6}$$

The general solution of (6) is

$$\begin{aligned}\rho_0&=\left(A-\frac{\sigma}{2m^2}\right)e^{mx}+\left(B-\frac{\sigma}{2m^2}\right)e^{-mx}+\frac{\sigma}{m^2}\\&=A'e^{mx}+B'e^{-mx}+C\,,\end{aligned}$$

where A and B are the integration constants respectively. Then assuming the solution of (5) in the form:

$$\rho=\rho_0+\varepsilon(x)\,,$$

and taking the relation of $\rho_0-m^2\rho_0+\rho=0$ and $\lambda ax\varepsilon/D=4.5\times10^{-10}\times x\varepsilon\ll1$ into consideration, the following relation will be obtained:

$$\frac{d^2\varepsilon}{dx^2}-m^2\varepsilon=-\frac{\lambda\rho_0 a}{D}x\,. \tag{7}$$

The general solution of (7) is

$$\begin{aligned}\varepsilon=&\left(A''+\frac{\lambda a}{2m}\left[\frac{A'}{2m}x-\frac{A'}{4m^2}-\frac{C}{m^2}-\frac{B'}{4m^2}-\frac{A'x^2}{2}\right]\right)e^{mx}\\&+\left(B''+\frac{\lambda a}{2m}\left[\frac{B'}{2m}x+\frac{B'}{4m^2}+\frac{C}{m^2}+\frac{A'}{4m^2}+\frac{B'x^2}{2}\right]\right)e^{-mx}\\&+\frac{\lambda a\sigma}{m^4}x\,,\end{aligned} \tag{8}$$

where A'' and B'' are the integration constants respectively.

In the present estimation the following relations are tentatively adopted as the boundary condition:

$$\rho_0=\varepsilon=0 \qquad \text{at} \qquad x=0\,, \tag{9}$$

$$\rho_0'=\varepsilon'=0 \qquad \text{at} \qquad x=h\,. \tag{10}$$

Namely, the relation of (9) is the boundary condition at the ground surface, and it holds in approximation as the account in the free air is in the ratio of $1:10^4\sim10^5$ with that in the underground. And the relation of (10) rightly holds in case of the existence of impervious rock layer for any gases at the depth of h from the ground surface. In practical cases the relation of (10) contains some difficulties, but in the present rough estimation of radon-density in the underground it is tentatively used

as a convenience for calculation. It is to be remarked that relation of (10) is approximately fitting for cases where the underground layer at the h depth is saturated with underground water. By applying the boundary conditions of (9) and (10) to the value of ρ_0 and ε, and calculating in numerical values with some omissions,

$$\rho_0 \cong \frac{\sigma}{m^2}(1-e^{-mx}), \tag{11}$$

$$\varepsilon \cong \frac{1}{D}\left(\frac{\lambda a\sigma}{m^4}x - \frac{\lambda a\sigma}{4m^3}x^2 e^{-mx}\right). \tag{12}$$

From the complete expression of $\rho=\rho_0+\varepsilon$, we have

$$\begin{aligned}\rho &= \frac{\sigma}{m^2}(1-e^{-mx}) + \frac{\lambda a\sigma}{Dm^4}x - \frac{\lambda a\sigma}{4Dm^3}x^2 e^{-mx} \\ &= \frac{Q}{Dnm^2}\left[(1-e^{-mx}) + \frac{\lambda a}{Dm^2}x - \frac{\lambda a}{4Dm}x^2 e^{-mx}\right].\end{aligned} \tag{13}$$

Applying the numerical values to the quantities in (13), and putting the numerical values of m as nearly 5×10^{-3},

$$\rho = \frac{Q}{Dnm^2}\left[(1-e^{-mx}) + 1.8\times10^{-5}x - 2\times10^{-7}x^2 e^{-mx}\right]. \tag{14}$$

For the purpose of examining the correctness of expression (13), the values of ρ at the depth of 0.5 m, 1.0 m, and 2.0 m below the ground surface are calculated in case of the underground temperatures of 20°C and 50°C at a place of 40 m depth, by applying the numerical values of practical use to the quantities in the expression (14). Namely, in this case, the density of the soil and the mass of radium in the soil of 1 cc. are assumed to be 2.0 and 5×10^{-12} g. cc^{-1} respectively. The amount of radon radiated from the radium in 1cc-soil is estimated to be nearly 6.9×10^{-23} g. cc^{-1} sec^{-1}. And from the calculation by I. Iwasaki (28), the amount of radon derived from a radium deposit exposed to air or to the coexistence of air and water is reduced to the amount in 8% of the total value of radon radiated. Taking the porosity as ($n=$) 0.4, the value of Q above defined is calculated as $Q=9.2\times10^{-19}$ c cc^{-1} sec^{-1}.

From these, we have, when the underground temperature is 20°C at a 40m-depth,

$$\rho_{50\,cm}=0.28\times10^{-12}\ c\ cc^{-1},\qquad \rho_{100\,cm}=0.52\times10^{-12}\ c\ cc^{-1},$$
$$\rho_{200\,cm}=0.82\times10^{-12}\ c\ cc^{-1};$$

and when the underground temperature is 50°C at a 40m-depth,

$$\rho_{50\,cm}=0.28\times10^{-12}\ c\ cc^{-1},\qquad \rho_{100\,cm}=0.52\times10^{-12}\ c\ cc^{-1},$$
$$\rho_{200\,cm}=0.82\times10^{-12}\ c\ cc^{-1},$$

where $\rho_{50\,cm}$, $\rho_{100\,cm}$ and $\rho_{200\,cm}$ are the underground radon-densities calculated at the

depth of 50 cm, 100 cm, and 200 cm respectively below the ground surface. These values are in good agreement, in the general aspect. with those observed by Z. Hatsuda (**26**) at the depth of 0.6 m, 1.0 m and 2.0 m below the ground surface in the yard of the Kyoto Sectional Meteorological Observatory, and those observed by E. M. Kovach (**27**) at the depth of 0.25 m, 0.75 m, and 2.0 m in the yard of Fordham University in New York. The value measured by Hatsuda at the depth of 2.0 m is practically 1.8×10^{-12} c cc^{-1} sec^{-1}., but the difference between the observed and calculated values in such an order of magnitude is easily eliminated by a little adjustment of the numerical values above used in calculation. In such a manner the underground radon-density at various depths are represented in Fig. 33 by a comparison of the calculated values with the observed values, and from the good agreement of both the calculated and observed values in a general aspect of underground radon distribution the correctness of expression (13) is considered to be proved to a certain degree.

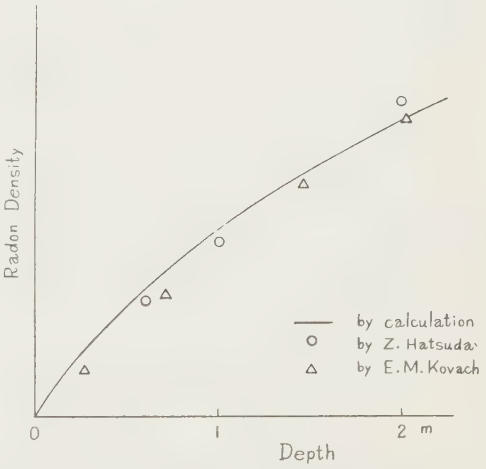

Fig. 33. Relation between the radon-density and the depth.

Concerning the components contributing to the ground radioactivity measured, are presumed the contributions by cosmic rays, the radiation from the radioactive substances in the atmosphere, radium, thorium and other elements in the soil, and the decay products of radon and thoron in the soil. But, in the present case in treatment on the problem of difference of ground radioactivity possibly by the difference of ground temperature, only the decay products of radon may effectively be disscussed, the others being reasonably excluded from the reason of their long life-time and other factors.

On the other hand, the value expressed in (13) is the radon-density in the gap of the soil, but the decay products in the underground water also contribute to the ground radioactivity and this should be calculated for a discussion on the difference of ground radioactivity due to the difference of underground temperature. Denoting the partition rate of radon to the underground water by S, S being assumed to be a function of x, the total ground radioactivity (I) originated from the underground radon is expressed as

$$I=\int_0^\infty \rho(1+S)\, Z\, e^{-\mu x}dx\,, \tag{15}$$

where μ is the absorption coefficient of radioactive rays, which shows practically different values with respect to the kind of rays, but in this case it is simply assumed as a constant, and Z is the rate of radioactive intensity being measured on the ground surface and originating from the β- and γ-rays radiated by the decay products of underground radon. But, since the radioactivity originated from the decay products of radon which is more deeply seated than 50 cm, cannot reach to the counter on the ground surface, it becomes

$$\int_0^{\infty} \rho Z e^{-\mu x} dx \cong \int_0^{50} \rho Z e^{-\mu x} dx .$$

Now, S can be assumed to be a constant, as far as the above-mentioned case concerns, from the reason that the temperature on the ground surface (about 50 cm depth) is everywhere the same, and consequently

$$I = \int_0^{\infty} \rho(1+S) Z e^{-\mu x} dx \cong \int_0^{50} \rho(1+S) Z e^{-\mu x} dx .$$

Introducing the expression (13) into the above expression, we get

$$I \cong Z(1+S) \frac{Q}{Dnm} \left[\frac{1}{\mu^2} + \frac{\lambda a}{Dm^3} \left(\frac{1}{\mu^2} \right) - \frac{\lambda a}{4Dm^2} \left(\frac{2}{(m+\mu)^3} \right) \right]. \tag{16}$$

In this case, assuming the value of μ roughly as 1.0, μ being measured as in the range value in 0.12~2.30 for the γ-rays of RaB~RaD in aluminum-absorption, and calculating I on the ground surface under the condition of ground temperatures of 50°C and 20°C at the 40m-depth respectively, we have

$$I_{50°} = Z(1+S) \frac{Q}{Dnm} (1+3.6\times10^{-3}-4\times10^{-6}) , \tag{17}$$

$$I_{20°} = Z(1+S) \frac{Q}{Dnm} (1+5\times10^{-4}-5\times10^{-7}) . \tag{18}$$

The values of I derived from both the expressions of (17) and (18) are considered to be nearly the same, and moreover the ground radioactivity measured at the ground surface contains various other kinds of radioactive rays besides the above calculated I. From these reasons, it is provisionally concluded that the observed fact that the relation exists between the strong ground radioactivity and the high underground temperature is not explained by a difference of ground radioactive intensity influenced only by the difference of underground temperature under such a process of reasoning as above treated. But the above calculated values are obtained from the omission and rough estimation of many quantities in the equations concerned, and, in case of the estimation of other values for D, for example, the difference between $I_{50°}$ and $I_{20°}$ may become a fairly large value deserving of discussion. Taking the small value of 0.00007 cm^2 sec^{-1} as the D of diffusion coefficient of underground radon (the diffusion

coefficient of radon in water at 18°C is 0.00001 cm^2 sec^{-1}) the values of I become

$$I_{50°}=Z(1+S)\frac{Q}{Dnm}(1+3.6-4\times10^{-3}),$$

$$I_{20°}=Z(1+S)\frac{Q}{Dnm}(1+0.5-6\times10^{-4}),$$

and these values of I seem to be in good agreement with the above considerations. From these reasonings the above-described explanation of strong ground radioactivity by the influence of high underground temperature or the application of the expression (16) to the problem will reasonably be reserved for a future development in the study of ground radioactivity in hot spring areas.

(*b*) *On other causes*

As other causes of the phenomena of strong ground radioactivity at places of high ground temperatures in hot spring areas, the following three are considered to be worth mentioning.

(i) Radon emanating from mobile underground water at places of high underground temperatures

In considering the systems of mobile underground water in hot spring areas, the ground radioactivity in hot spring sites in the upper course of a stream of mobile underground water is generally larger than that in the lower course. This tendency was observed in the hot spring areas at Misasa, Hamamura, Kinosaki, Akune, Yuroura and others as shown in the respective figures. This is interpreted as being a process where the quantity of radon dissolved in the underground water at a place of low underground temperature is nearly 30% of the total radon-content at that point, and that at higt temperature zone this is reduced to nearly 10% of the total radon-content according to the change of partition rate, as 20% of the total radon-content emanates from the underground water in the movement of underground water containing radon from the low to the high temperature zones of underground layer. And the value of ground radioactivity is again reduced to the normal value in the movement of the underground water from the upper course of high underground temperature to the lower course of low underground temperature by the consumption of radon to supply the lack of dissolved radon in the underground water. As these emanations of radon contained in the underground water in the high temperature zone gradually take place in the course of underground water movement, they are only considered to contribute partly to the large ground radioactivity in the hot spring area, and we do not presume them to be the most influential causes. It is to be remarked that the ground radioactivity measured at the ground surface may, under favourable circumstances, become

an object of research in the observation of mobile underground water and the distribution of underground temperatures along the stream.

(ii) The difference of emanation rate of radon from the rock or soil according to the difference of temperature

The fact was reported by I. Iwasaki (28) that the higher the temperature, the stronger the radon emanation from the rock or soil containing radium. And, in the preceding section, the place at $x=h$ was layered by impervious bed rock, but, instead of bed rock, a place filled with underground water could also be assumed. In such a case, the radon in the hot underground water rises easily up to the ground surface in the form of bubbles compared with that in the cold underground water. Consequently, the combined effects of the increase of radon emanation from the rock or soil at the place of high underground temperature and the increase of mobility of radon ascending through the underground layer up to the ground surface, also influenced by the high underground temperature, are provisionally expected to explain the relation between the strong ground radioactivity at a hot spring site.

(iii) Precipitation of radium along the stream of thermal water of hot springs

As hot water dissolves a larger amount of radium compared with cold water, the hot water dissolves the large amount of radium contained in the rock in the deep and hot underground layer along its stream course, and precipitates the dissolved radium at the ground surface of comparative low temperature, mixed by the cold underground water. By this reasoning, the strong ground radioactivity at the hot spring site is partly explained.

In concluding this section, an explanation of the relation between the strong ground radioactivity and the hot spring site, compared with other points in the same area was considered. The intimate correlation between the strong ground radioactivity and the point of high underground temperature found in the present observation seems especially to offer a solution to this problem. On this point the intensity of ground radioactivity to be observed on the ground surface was theoretically calculated from the solution of the diffusion equation of radon in the underground layer in both cases of high and low underground temperatures. The numerical results thus obtained were not successful in explaining the above-described relation, but its conclusion may properly be postponed for future investigation, when the quantities roughly estimated in the present treatment are more accurately measured and rightly applied to the calculation. Three other probable causes of the relation between hot spring sites and strong ground radioactivity were also discussed. Of these, the most promising as-

sumption was supposed to be the strong emanation of radon from the rock or soil containing radium under high temperature conditions, having the increased effect of easy upward liberation of radon through the underground layer to the ground surfaces. Concerning these problems, model experiments using radioactive isotopes will eagerly be recommended and urgently be commenced in a succeeding research.

5. On the depth of the radon supply-source

The problems on the process of absorption of radon by the thermal water of hot springs and on the depth of the radon supply-source have been studied by many researchers such as K. Kuroda (**29**), I. Yokoyama (**30**) and recently by K. Kikkawa (**31**). In the present investigation a model on the process of radon-absorption by the hot underground water was tentatively proposed to fit the observed facts, as in (c) of § 2. Judging from the observed data, that the age of radon measured in the thermal water on the ground surface is estimated to be 20~170 minutes after the absorption by the thermal water ass hown in Figs. 5~8, the supply-source of radon, which is measured in the thermal water on the ground surface and problematically treated in the present investigation, was reasonably presumed to lie in a comparatively shallow underground layer.

The assumed shallow origin of radon-supply is ascertained by the other observed results, that the values of the ratio of ground radioactivity at the hot spring site to that of a place remote from the hot spring in the same area are nearly equal to each other in any case of radioactive or non-radioactive hot springs, as shown in Table 1. In the area of Misasa hot spring, for example, the absolute values of ground radioactivity are generally larger compared with those in other hot spring areas as shown in Table 2, but the said values in Misasa are nearly the same as those in other areas of non-radioactive springs. These observed data tell us that the supply-source of radioactivity lie in comparatively shallow layers and do not coincide with the deep original source of the hot spring. Should the deep origin of radioactivity coincide with the origin of hot water, the ratio between the ground radioactivity at the radioactive hot spring site and that at a remote point should show a larger value than those obtained in the practical measurements. The reason for the strong radioactivity of the thermal water in the Misasa hot spring area is believed to be that it happens to lie and gush out the hot water in an area of strong ground radioactivity, and moreover there are many holes in the underground of this area which contain enriched radium deposited by the underground water. These are ascertained by the observed data on the radon measurement of underground water in the area of Misasa by K. Sugihara (**32**) that the radon-content in the underground water shows a large value which is nearly the same as that of hot spring thermal water. From these it is

reasonably concluded that the source of radon-supply generally lies in a shallow underground layer in the area of the hot spring and has no connection with the original source of the hot spring thermal water.

Table 1. The ratio of intensity of the ground radioactivity at the places near hot spring to that at the remote places from hot springs.

Name	Radon-content (mache)	Mean value at places near hot springs. (A) Figures in brackets show the number of examples.	Mean value at places remote from hot springs. (B) Figures in brackets show the number of examples.	The value of (A/B)
Misasa (north)	30–200	1.17 (9)	1.09 (9)	1.07
Misasa (south)	10–100	1.39 (8)	1.21 (16)	1.15
Togo	2–8	1.18 (9)	1.05 (7)	1.12
Asozu	1–3	1.05 (5)	1.09 (4)	0.96
Hamamura, Kachimi	3–10 (50)	1.37 (9)	1.19 (11)	1.15
Kaike	0–0.5	1.48 (2)	1.27 (14)	1.17
Yoshioka	1–3	1.32 (9)	1.19 (18)	1.11
Iwai	0–2	1.18 (8)	1.03 (15)	1.14
Yudani	4–7	1.36 (5)	1.14 (22)	1.19
Kinosaki	5–12	1.41 (13)	1.23 (19)	1.14
Yumura	0–2	1.31 (9)	1.23 (18)	1.07
Akune	5–12	1.34 (9)	1.21 (23)	1.11
Yunoura	—	1.43 (9)	1.28 (20)	1.11

Table 2. Relation between radon content and number of counts. (measured with the same counter tube)

Name	Radon-content (mache)	Number of counts near the springs. (c.p.m.)	Name	Radon-content (mache)	Number of counts near the springs. (c.p.m.)
Misasa	180	106±3	Asozu	2	70±2
	120	90±3		2	73±2
	20	88±3	Kachimi	10	72±2
	22	87±3		3	85±3
	4	88±3		48	90±3
	80	99±3	Hamamura	2	65±2
Togo	4	83±3		3	60±2
	7	71±2			
	3	64±2			

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ELASTIC PROPERTIES OF ROCKS WITH RELATION TO THE EARTH'S INTERIOR

BY

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ABSTRACT

With respect to the value of the pressure coefficient of rock-elasticity hitherto discussed, the remarkable discrepancy between the values obtained from rock-experiments and from the velocity distribution of seismic waves in the Earth's interior was fairly reconciled by an analysis of the rock-experiment data by considering the relation between the elasticity and density rather than the direct treatment of the pressure.

The pressure coefficient of the bulk modulus and the velocity of the dilatational elastic wave with respect to dunite determined from the high pressure experiment was ascertained to be in a good agreement with those obtained from the velocity distribution of the seismic wave in the upper mantle. In the present article particularly the effect caused by micro-pore space in the rocks upon the character of rock-elasticity in a range of comparatively low pressure is discussed in detail.

1. Introduction

The study of the structure and property of the Earth's interior is an important and attractive field of research in geophysics. Recent developments in geophysical techniques have greatly facilitated the observational approach even to the deep central part of the Earth. The Earth structure with respect to the seismic wave velocity has been investigated more and more in detail by the analysis of seismograms obtained at many well-equipped observatories all over the world. And, moreover, our knowledge on the elastic property of the Earth has also gradually advanced with that of the phenomena of Earth tides and latitude variation. Concerning the density distribution of the Earth's interior the values as derived from various theoretical and observational treatments have shown a fairly good agreement in recent years. But problem of temperature distribution and the thermal state of the Earth's interior still remains unsolved due to the lack of knowledge on the internal distribution of radioactive elements and to other reasons as well. Even if the structure and such properties of the Earth's interior as the existence of discontinuity and the physical quantities of density, pressure, temperature, elasticity, and seismic wave velocity were fully clarified, another important problem may yet remain, i.e., of what is the Earth's interior composed and how does

this core material under high pressure and temperature behave against both internal and external stress? In this point ordinary geophysical field observation is generally powerless for examining the internal dynamics of the Earth. Laboratory experiments and theoretical considerations of materials under high pressure and temperature, are certainly powerful means for this type of investigation especially on the phenomena of plasticity and rupture in the Earth. The properties of plasticity and the modes of rupture of the material in the Earth may doubtlessly play an important role in the process of earthquake occurrence, accumulation of seismic energy, crustal deformation, orogeny and epirogeny, continental drift, core mobility and other important telluric phenomena.

A great advance in the experimental study of the effect of high pressure and temperature on property changes in rocks has been made by the elaborate experiments in U. S. A., such as those of L. H. Adams at the Geophysical Laboratory, and P. W. Bridgman and F. Birch at Harvard University. The compressibility of rocks have early been measured up to some tens of thousands bars of pressure under several hundreds degrees centigrade, but it is only recently that the velocity of elastic waves in rocks could be measured by the supersonic method up to only several thousands bars under several hundreds degrees as was done by D. S. Hughes and J. H. Cross at the University of Texas. As the pressure of 10^3 bars, 10^4 bars, and 10^5 bars correspond roughly to those prevailing at the depths of 3km, 30km, and 300km respectively below the Earth's surface, their respective pressure values tell us also the pressure state at the foci of local shallow earthquakes, large destructive earthquakes, and deep plutonic earthquakes. For the purpose of studying the nature and occurrence mechanism of large destructive earthquakes it is desirable to examine by laboratory experiment the change in rock properties under conditions of more than several ten thousands bars of pressure and one or two thousands degrees centigrade, and property to be examined should contain the density change, elastic property, seismic velocity, thermodynamical element, electric and magnetic property, plastic deformation and mechanism of rupture. These demands will be fulfilled as experimental technique advances. But the artificial arrangement of rocks in the laboratory in the state of more than several hundreds of thousand or several million bars of pressure and several thousands degrees centigrade to correspond to the state of material in the lower mantle and core of the Earth's deep interior is technically almost, if not quite, impossible in spite of the astonishing advance of modern scientific techniques. On the other hand, the behaviour of materials under these conditions of very high pressure and temperature such as more than several tens of millions bars of pressure and degrees of temperature can be rather simply and easily treated from the theoretical side of modern thermodynamics, namely for study on the internal state and energetics of the Sun and other stars, the optically and magnetically

variables, the pulsating stars, super-novae and other astrophysical phenomena, the purely theoretical treatment can be and has been successfully applied. From the circumstances above stated it may be concluded, if we may be permitted an over-simplification, that pressure and temperature in the Earth's deep interior are too high to be experimented in the laboratory and too low to be treated by the purely theoretical thermodynamics of recent development. Therefore, a combined method is naturally introduced for investigating the nature and behaviour of the material, that is a method of extrapolation to a higher position of the experimental data obtained at the lower pressures and temperatures or to a lower position of the theoretical deductions of modern thermodynamics for the higher pressures and temperatures along with a suitable method of combining the results.

Fortunately, the values of some physical quantities such as density, pressure, seismic wave velocity and elasticity, even in the deeper part of the Earth can be fairly accurately determined by various geophysical observations on the Earth's surface, so that the validity of prospection assumed in various theoretical treatments of the material in the Earth's deep interior can safely be checked by comparing these theoretically deduced values with those of physical properties directly observable. However, we should not confine our studies to this aspect alone or remain satisfied with a good agreement between theoretically derived geophysical values and those obtained from geophysical field observation, but we should advance towards the ideal of deriving a theory and model of matter which would explain the reason for discontinuity and the mode of behaviour of material in the Earth's interior under high pressure and temperature against the internal stress and external force. After these considerations the nature of earthquakes and their source of energy, the existence of the central core of the Earth and its mobile property, and even the original formation of the Earth itself, may possibly be revealed to us in process of time.

Concerning studies hitherto made on the Earth's interior along the lines above stated, i.e., the application of the modern theory of matter to the material in the Earth under high pressures and temperatures in consideration of data based on experimentation and direct geophysical observation, a brief account will be given: Early in 1923 E. D. Williamson and L. H. Adams (**1**) and others discussed the density distribution of the Earth's interior by taking the pressure dependency of density and compressibility of rock into account, while, on the other hand, H. Haalck (**2**) introduced, for the first time, the concept of atomic potential in a simple form of interatomic repulsive force into the calculation of the relation between the compressibility and pressure of the material in the Earth. And these two pioneering works, owing to their different standpoints, are also considered to represent the establishment of the two important and distinct ways of research on the Earth's interior according to the modern theore-

tical treatment of state of matter. Namely, the former is the extrapolation of rock properties obtained in laboratory experiment to the deeper part of the Earth, while the latter examines the validity of the theory postulated by comparing its deduced values with those derived from geophysical observation.

On the most attractive and essential problem of the Earth's core, W. Kuhn and A. Rittman (**3**) published in 1941 their opinion on the large hydrogen content in the core, a concept which differs greatly from the ordinary one. K. Sezawa and K. Kanai (**4**) treated the problem of the core in a thermodynamical way in 1940 and 1941, and several researchers such as J. Lynch (**5**), H. Lorent (**6**), A. Eucken (**7**), A. Mercier (**8**), have also taken up the problems of the core in various ways. Since 1948 W. H. Ramsey (**9**, **10**, **11**, **12**, **13**) treated the formation and existence of the core of the Earth and other planet from the very particular standpoint of the phase transition of matter by pressure only, and discussed the existential condition of the central core in each solar planet and the difference of their mean densities. In 1952 H. Miki compared (**14**) the degrees of stability between the three phases of solid, liquid and gas as being the state of the Earth's core from the standpoint of statistical mechanics. On the property of the mantle F. Birch applied since 1935 (**15**, **16**, **17**, **18**) the F. D. Murnaghan theory of finite strain to the Earth's interior with the consideration of date derived from laboratory experiments under high pressure, and treated various problems of the mantle. Y. Shimazu and H. Takeuchi also treated the problem of the Earth's interior since 1949 (**19**, **20**) according to the Murnaghan theory of finite strain, and Y. Shimazu (**21**, **22**) published comprehensive papers for study of state of materials composing the Earth's interior from the standpoints of the finite strain theory and the comparative method with meteorite constituents.

On the density distribution of the Earth's interior, K. E. Bullen has been very actively developing his theory since 1936 (**23**, **24**, **25**, **26**, **27**, **28**, **29**, **30**) and W. M. Elsasser (**31**), H. Miki (**32**), and others have also been treating the problem of density distribution from approach of statistical mechanics. In connection with Bullen's density distribution a brilliant work on the theory of Earth tides was made by H. Takeuchi in 1950 (**20**).

Concerning the problem of temperature distribution within the Earth, many studies have been made from various standpoints, and here the researchers directly connected with the Earth's interior may be mentioned. F. Birch (**33**), H. P. Coster (**34**), W. D. Urry (**35**), A. E. Benfield (**36**), E. C. Bullard (**37**), A. E. Verhoogen (**38**), R. J. Uffen (**39**), T. Rikitake (**40**), J. A. Jakobs (**41**, **42**), H. Miki (**43**) and others have been making great contributions to the study of thermal states within the Earth. Lastly, in the recent development in theories on the origin of the main geomagnetic field and its secular variation, we are greatly indebted to W. M. Elsasser (**44**, **45**, **46**, **47**) and

E. C. Bulland (**48**, **49**) whose theories were fairly well ascertained by H. Takeuchi and Y. Shimazu (**50**, **51**) from the standpoint of a self-exciting process in the magneto-hydrodynamics within the core. In connection with geomagnetism, the electric properties of the Earth's interior were, recently investigated in detail by T. Rikitake (**52**).

Under the circumstances above stated, the various problems concerning the state and nature of material in the Earth's interior will be taken in our series of investigations and will be treated from the theoretical point of view in consideration of the data obtained in laboratory experiments and geophysical field observations. In the present article the data obtained in laboratory experiments on the elastic properties of rock are discussed, comparing with those derived from the observation of natural earthquakes. And the conclusions are applied to the problem of the elastic state of the Earth's mantle, along with some considerations on its constitution.

2. Compressibility of rocks and the Earth's interior

The velocities of seismic waves in the Earth's interior have been investigated more and more in detail since the days of E. Wiechert (**53**); and, except in some particular points, the values recently obtained by H. Jeffreys (**54**), B. Gutenberg and C. F. Richter (**55**), and B. Gutenberg (**56**) generally show a fairly good agreement.

From the data obtained by the analysis of seismic waves, we can derive both density and the elastic constants at any point in the Earth, under the assumption that the material in the Earth is in a state of compression by the weight of the overlying material; that is,

$$\frac{d\rho}{dR}=\frac{\rho}{k}\frac{dP}{dR}, \tag{1}$$

$$\frac{dP}{dR}=-g\rho, \tag{2}$$

where P, k, ρ, and g are the pressure, bulk modulus, density and gravity acceleration respectively at a point of distance R from the Earth's centre.

The above relations were first applied by L. H. Adams and E. D. Williamson (**1**) and recently by K. E. Bullen (**28**).

And k/ρ in equation (1) is, on the other hand, expressed by the velocities of the seismic P and S waves as follows:

$$V_p=\sqrt{\frac{k+4\mu/3}{\rho}}, \tag{3}$$

$$V_s=\sqrt{\frac{\mu}{\rho}}, \tag{4}$$

$$\frac{k}{\rho}=V_p^2-\frac{4}{3}V_s^2, \tag{5}$$

where V_p, V_s, and μ denote the velocity of the P wave, S wave and modulus of rigidity respectively. Therefore, the density, pressure and elastic constants at any depth in the Earth's interior will be expressed as some function of radius R when the temperature of the Earth is not high.

On the other hand, many laboratory experiments were made to examine the effect of pressure upon the elasticity of rocks and the results thus obtained were compared with those of the seismic data.

In this case, there always remains the problem as to what sort of rock each layer of crust and the mantle are composed of. From the geological standpoint, rock of the olivine system which contains a great quantity of Mg strongly combined with isolated SiO_4-tetrahedron is considered to form the lower part of the Earth's crust, and, as the layers go up in the crust, rock abundant of SiO_4 compounds with Ca and Fe as well as pyroxene groups (mainly of di- and tri-silicates) are found to prevail. The upper part of the crust is generally composed of granitic rock containing Na and K. With regard to the material underlying the crust, some geologists insist on the existence of basaltic rock under the granitic crust, while others assert eclogite as the under crustal material. It is natural to suppose that the rock under the crust may be considerably deformed by pressure; therefore, the material under the crust may possibly be treated as in a pseudo-crystalline or vitreous, glassy state. On the constituents and system of rocks composing the layers of the crust and the upper part of the mantle some detailed treatment will be made in a succeeding article in relation to the geophysical (geodetic, seismic and meteoritic) investigations of the nature and behaviour of the crust.

In the present article the problem will be treated under the simple assumptions that the material of the crust is mainly granitic and gabbroic, and that underlying the crust is dunite. These assumptions are generally adopted in various aspects of seismology, geology, mineralogy and the study of meteorites. H. Jeffreys pointed out the fact in his paper in 1952 (**57**) that the increasing degree of seismic wave velocity observed in the upper mantle differs greatly from that deduced in laboratory experiments on the effects of high pressure on the property of rocks. Near the surface of the mantle at some 30 km depth and under the pressure of 10^4 bars the value of dk/dP calculated from the data of seismic wave velocity obtained by H. Jeffreys (**54**) and B. Gutenberg (**56**) are

$$\frac{dk}{dP}=4\sim6\,. \tag{6}$$

On the other hand, the value of dk/dP obtained from the laboratory experiments on dunite (mainly composed of olivine) by L. H. Adams (**58**) is as the mean value between the pressure condition of 2000~12,000 bars

$$\frac{dk}{dP}\approx 20\,. \tag{7}$$

This discrepancy is not attributable to the difference between the effect of temperature distribution usually assumed in the Earth's interior upon values obtained from the seismic waves and the effect of room temperature upon the laboratory experiment. Jeffreys suggested the following three effects as the possible cause of the discrepancy: (i) A rapid decrease in the increasing rate of elasticity with increasing pressure; (ii) An anomalous increase in the effect of temperature upon elasticity with depth; (iii) An increase in the degree of admixture of low velocity material with depth. In this article this discrepancy will be treated below.

On laboratory experiments on the effects of pressure and temperature on the elasticity of rock, many elaborate works have been made by L. H. Adams and his collaborators (**58**, **59**), Zisman (**60**), Ide (**61**), Bridgman (**62**, **63**), Birch and his collaborators (**16**, **17**, **18**), and Hughes (**64**, **65**). Through their experiments, many important properties of rock under high pressure have gradually been found, such properties as the rapid increase in rock-elasticity by pressures up to 2,000 bars and over that pressure a somewhat slow increase, and a hysteresis phenomenon of rock deformation under a low pressure. Zisman examined the difference of property in direction, the hysteresis effect and the relation between the elastic constant and the frequency of applied force. Ide reported a considerable effect of hysteresis in his experiment on the change of dilatational velocity with temperature. Recently Hughes and Cross (**65**), investigated the rock-elasticity by applying the supersonic wave in the range of up to 5,000 bars-pressure and 300°C-temperature. In his experiment it was clearly observed that the mode of change of elastic wave velocity with pressure and temperature is classified into two parts. Namely, the velocity of elastic wave of rock increases rapidly with pressure up to nearly 2,000 bars and then slowly and linearly increases in the pressure range over 2,000 bars as seen in Fig. 1.

They explained the rapid increase of wave velocity in the low pressure range as derived from the effect of a compression of the pore spaces in rock and the slow and linear increase in high pressure range as the normal and reversible change of wave velocity with pressure. In his paper it is reported that the frequency effect above described is negligibly small.

It is easily understood from the above-mentioned complexity of rock under high pressure and temperature that it is a very difficult task to compare the data obtained by laboratory experiments with seismometric and other observational data from the Earth's interior. In other words, the application and extension of the knowledge on rocks derived from laboratory experiments to the interpretation of any property of the real existing matter in the Earth's deep interior should cautiously be made under the consideration of difference of structure, chemical composition, physical state and other various circumstances in both cases.

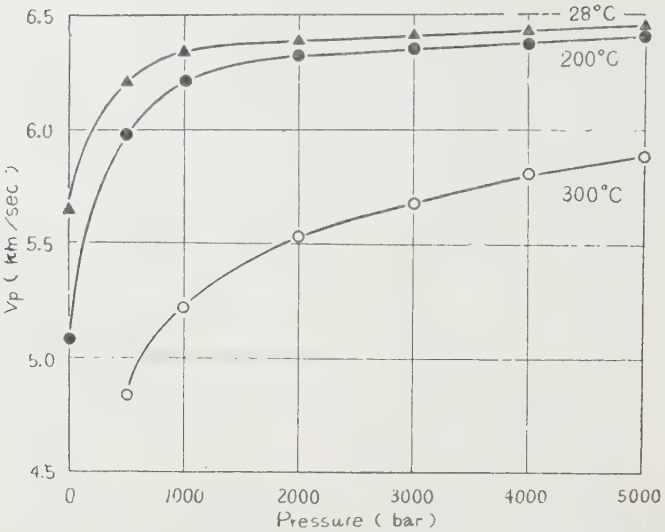

Fig. 1a. Relation between the dilatational wave velocity of granite and the exerted pressure. (After Hughes and Cross)

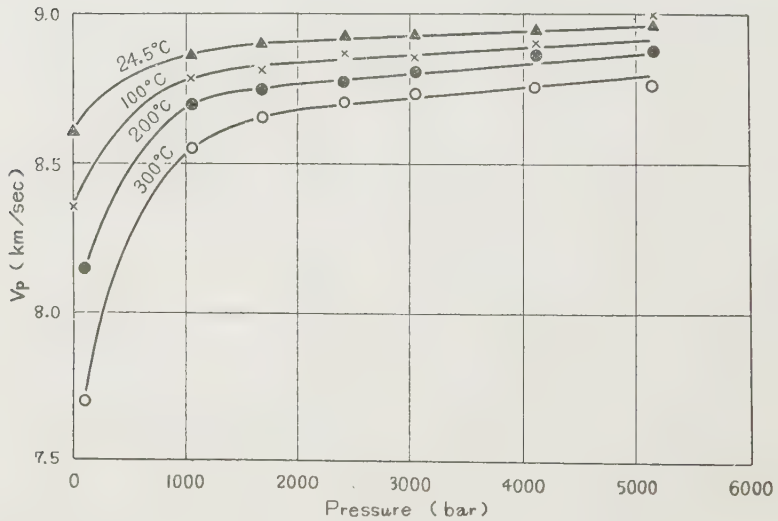

Fig. 1b. Relation between the dilatational wave velocity of dunite and the exerted pressure. (After Hughes and Cross)

From Fig. 1b, we see that at high pressure $dV_p/dP \approx 0.2$ km/sec·10^4 bars for dunite, whereas from the data by seismic observation, we find $dV_p/dP \approx 0.1$ km/sec·10^4 bars at the depth of 300 km (Jeffreys and Gutenberg). Therefore the discrepancy pointed out by Jeffreys was almost disappeared. But, as D. S. Hugdes and J. H. Cross pointed out, there still remains the effect of porosity. In the present article,

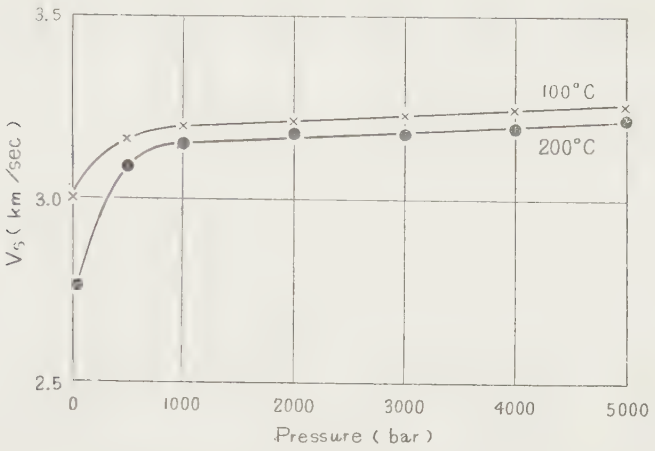

Fig. 1c. Relation between the shear wave velocity of granite and the exerted pressure. (After Hughes and Cross)

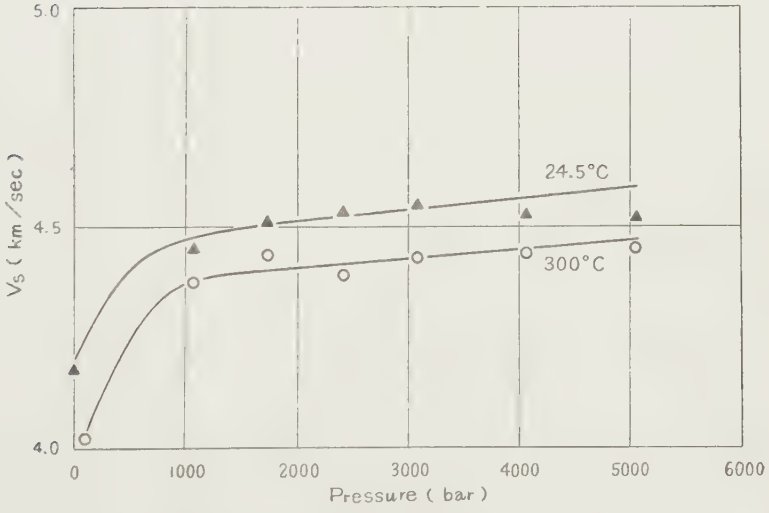

Fig. 1d. Relation between the shear wave velocity of dunite and the exerted pressure. (After Hughes and Cross)

the writer tries to examine the variation of elastic constants due to pressure only, eliminating the effect of porosity and temperature.

Among the various conditions affecting the elasticity of rock, the existence of porosity is generally considered and experimentally ascertained as one of the most essential. The pore spaces in rock are considered to originate from the circumstances in the epoch of rock formation. Namely, of the sedimentary rock, a considerable amount of pore spaces is inevitable as the sedimentary rock is formed under very low pressure, and also the igneous rock may have derived from the ejection and pool of gas con-

tained in the liquid state during the period of rock-solidification. Even in a single crystal of natural quartz many cracks and pore spaces are frequently found. Thus all rocks are generally considered to be more or less porous material and their elasticity depends largely on the amount and manner of distribution of porosity. In the process of rock compression under increasing hydrostatical pressure, the pore space would, at first, become deformed and the main substance would suffer little compression. Therefore, the compressibility of rock measured in the laboratory is considered to show apparently a considerably large value in the range of initial low pressure and gradually to approach the small real value after the stage of pore space disappearance. These effects are shown in Hughes' experiment on the variation of elastic wave velocity with pressure and, in his case, the effect of pore space was estimated to be negligibly small over the range of pressure of 2,000 bars. And, on the other hand, what are the practical effects of pore space on the elastic property of the material in the Earth's interior? To what depth of layer in the Earth is estimated the existence of influential pore space? As a natural consequence of Hughes' and other laboratory experiments, the effect of pore space upon various properties of rock is roughly considered to be influential upon the crustal layer down to nearly 5 km in hydrostatic pressure of 2,000 bars, and ineffective for the deeper part of crust and mantle. But, from another point of view, the enclosed volatile constituents in the deep crustal layer and the upper mantle may exert the same influence as the pore space upon various properties of the material in the Earth's interior. This effect is moreover complicated by the condition of high temperature, and renders the problem of the properties of material in the Earth more intricate.

Next, the phenomena of hysteresis observed in the laboratory experiment of rock will be mentioned. Ide and Zisman reported the conclusion that the hysteresis of rock under changes in pressure of up to 2,000 bars and temperatures within several hundreds degrees may mainly be attributable to the existence of pore space, contrary to the normal hysteresis caused by the so-called plastic flow. The deformation by pressure of cavity artificially prepared in glass and granite was experimentally examined by Bridgman (**63**) and a certain kind of rupture was observed around the cavity. The same kind of rupture of micro-cracks as in Bridgman's case would probably be found around the pore spaces in rock in the high pressure experiment. On this point the following rough estimation may be useful. From the laboratory experiment, the amount of pore spaces in rock is ascertained to be greatly decreased at the pressure of some 2,000 bars. And then the deformation of rock cavities is calculated from the standpoint of the theory of perfect elasticity. In this case the mutual distance of small cavities in the perfectly elastic rock is assumed to be very much large compared with the radius of cavity. And then a spherical surface of suitably large radius which

contains only one cavity at its centre can easily be imagined. And the pressure exerted on the spherical surface thus imagined is safely assumed to be the same as that hydrostatically applied to the rock-sample. In this case the radial displacement u at the point of distance r from the centre in the sphere is given by the following (cf. A. E. H. Love (**66**)):

$$u=\frac{r_0^3 p_0}{(3\lambda+2\mu)(r_i^3-r_0^3)}r+\frac{r_i^3 r_0^3 p_0}{4\mu(r_i^3-r_0^3)}\frac{1}{r^2}, \qquad (8)$$

where p_0 denotes the external pressure, λ, μ Lame's elastic constants, r_i and r_0 the radius of cavity and radius of the sphere respectively. In case when $r_0 \gg r_i$, the terms higher than $(r_i/r_0)^2$ can be neglected in a first approximation, and in this case the volume change of cavity does not depend upon the value of the radius. Taking the bulk modulus and rigidity of rock as some 10^{12} and 5.10^{11} c.g.s., the volume change of cavity by the pressure of 2,000 bars is estimated to be about 2 percent of the original volume of cavity. Under these circumstances the pressure of some 10^5 bars should be applied for reducing the cavity-volume to one half. From these calculations it is safely concluded that the formation of small ruptures and micro-cracks around the pore spaces in rock play an important role in the process of compression by high pressure and the phenomena of hysteresis in the laboratory experiment, especially within the range of pressure up to some 2,000 bars.

Concerning the hysteresis of rock caused by temperature-change, Ide (**61**) proposed a theory that the hysteresis of rock is caused by the formation of pore spaces in rock due to the directional difference of thermal expansion of constituent minerals. From Ide's interpretation and the above-mentioned consideration, the hysteresis of rock under the change of pressure and temperature should generally be related to the formation and deformation of pore spaces in rock in the experimental range of pressures up to 2,000 bars and temperatures up to 300°C. Irreversible phenomena other than hysteresis caused by the effect of viscosity, etc., have not yet been investigated in detail in laboratory experiments and these are necessary for clarifying the nature and behaviour of rock under high pressure and temperature. On the other hand, the property of irreversibility of the material in the Earth's interior has been studied by some researchers and it was reported by H. Jeffreys (**54**) from the study on the attenuation of energy of seismic waves that the irreversibility phenomena were scarcely observed in the mantle in case of wave propagation of short period such as seismic waves. Therefore, also on the problem of hysteresis and other irreversible phenomena, the direct application of the results obtained in the laboratory experiment of rock under high pressure and temperature to the interpretation of the nature and behaviour of material in the Earth's interior should be made more and more cautiously.

Next, with regard to the important problem of effect of the frequency of the applied

force upon the value of rock properties measured in the laboratory experiment, the measurement on changes of rock properties by applying a force of variable frequency on the same sample of rock could hardly be found, especially under the conditions of varying pressures and temperatures. It is very important and essential in the investigation of the nature and behaviour of materials in the Earth's interior to clarify the difference of rock properties measured by, for example, supersonic waves, natural seismic waves, static or long continuing and one-directional forces, and to operate the experiment under high pressures and temperatures. Such experiments are urgently needed, for the development of Earth science.

As above stated there remain many important and critical unsolved problems such as the hysteresis, the frequency-effect, and others in the research field of the application of rock-experiment results to the Earth's interior. But in the present treatment we shall confine ourselves to the effect of pore space upon the elasticity of rock and put aside the other effects until they are fully measured by laboratory experiments in a near future.

From the standpoint of statistical mechanics in the theory of solids, the bulk modulus of single crystals is generally supposed to be expressed by two terms. One term is derived from the mutual action of atoms at their mean position, and the other from the vibration in thermal agitation of atoms around their mean positions. The former depends only on the specific volume of substance, and the latter on both temperature and specific volume. The contribution of the former to the bulk modulus of solids is much larger than the latter at low temperatures such as room temperature, and under such circumstances, the bulk modulus of a single crystal depends only upon its specific volume and is independent of its temperature. Of the polycrystalline substances, the effect upon the bulk modulus by the mosaic structure should be added to the above-described terms. In this case, as the effect by the mosaic structure, the effects derived from the directionality of constituent single crystals, the impurity at the grain boundary, the lattice-defect and other will be considered. These effects are supposed to have little influence upon the bulk modulus of such substances as the metal, for example, whose structure is undirectional and compact in grain-boundary. But in the case of rocks of complex structure, the effect of the mosaic structure should not be neglected. These effects are considered to be also related, in an approximation, to the specific volumes of rocks as deduced from their energetical consideration if not irrevesible.

The effect of the existence of pore space upon the bulk modulus of rock has certainly no simple relation to the specific volume of the rock as the changes of pore space comprise an irreversible process. Therefore, from the reasoning mentioned above it is possible to state that the effect of temperature and pore space upon the bulk modulus of rocks which are supposed to be very complex in nature will certainly be

investigated by analysing the deviation from the normal and simple relation of the bulk modulus with its specific volume obtained from laboratory experiment. Under these assumptions, the relations between the bulk modulus and density (reciprocal of specific volume) were investigated with the data of elastic wave velocity of granite and dunite obtained from laboratory experiments under high pressures and temperatures by D. S. Hughes and J. H. Cross (**65**). In Fig. 2 the relations of the bulk modulus with the density are shown with respect to granite and dunite which were calculated from Hughes' results. It ought to be mentioned that the values of density and bulk modulus shown in this figure were calculated from the following relations:

$$\frac{dP}{d\rho}=\frac{k}{\rho}\,, \tag{9}$$

$$\frac{k}{\rho}=V_p^2-\frac{4}{3}V_s^2\,. \tag{10}$$

As in Hughes' paper only the values of pressure, temperature and their corresponding values of elastic wave velocity were specified, the values of ρ were obtained by calculating first the values of k/ρ from the values of V_p and V_s by (10) and then integrating equation (9). And the value of k was calculated by (10) for the known value of ρ. Moreover in the calculation the coefficients of the cubical thermal expansion (α) of rock at zero pressure were to be assumed and, in this case, the values of α of 2.4×10^{-5} and 2.7×10^{-5} were adopted for granite and dunite respectively (cf. Birch (**67**) and Landolt-Börnstein (**68**)). It should be remarked here that the above two equations (9) and (10) are safely applied to the case of isotropic and perfect elastic material, and its application to the real rock, especially to the porous and imperfect elastic rock under low pressure, should be made very cautiously. But in the present treatment, the above equations were tentatively and suitably used for the purpose of determining the pressure-limit of the effect of pore space for the rock-elasticity.

As seen in Fig. 2 the bulk moduli of granite at various temperatures are simple and linearly related to the density under high pressure. In granite the bulk modulus depends only on the density over the pressure of 2,000 bars. In dunite the curves of bulk modulus at various temperatures also have a common asymptotic line under the high pressure range of over 5,000 bars, and these asymptotic lines in both cases of granite and dunite are certainly supposed to express the behaviour of bulk modulus which is a function of only one parameter of density. Namely, taking the pressure of 2,000 bars for granite and 5,000 bars for dunite as a critical pressure, it is roughly stated that the behaviour of rock with respect to its bulk modulus may conventionally be divided into two parts, namely, the low pressure range affected mainly by pore

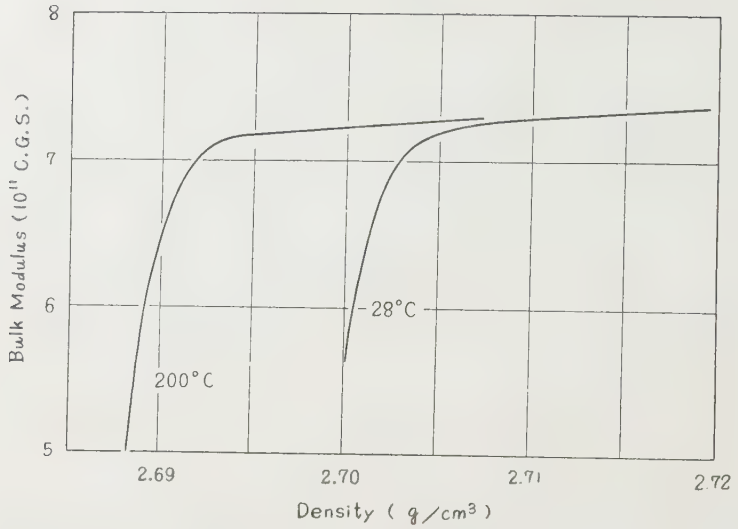

Fig. 2a. Relation between the bulk modulus and the density for granite.

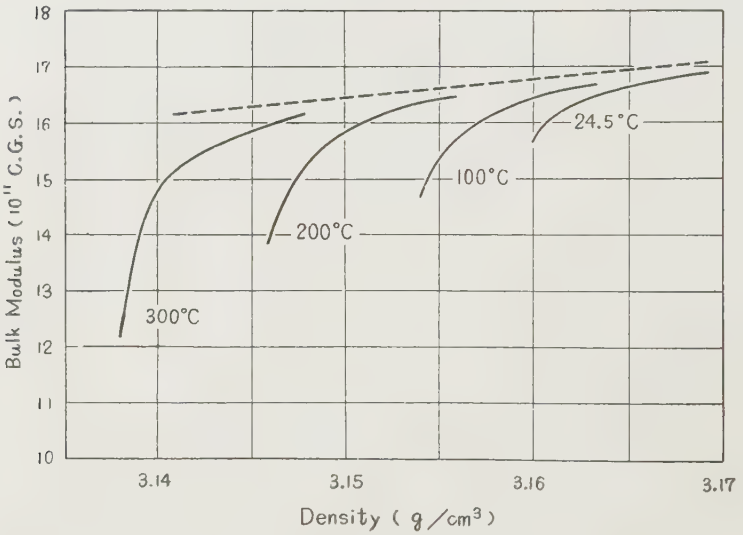

Fig. 2b. Relation between the bulk modulus and the density for dunite.

space and the high pressure range depending upon the interatomic potential. Practically, an asymptotic line will easily be found in the data over the pressure of 2,000 bars in the case of granite, but for dunite it is somewhat difficult to draw such a definite line as in the case of granite. In the case of dunite a line was tentatively drawn by connecting each value of k at their uppermost pressure (5,000 bars). From these considerations the values of $dk/d\rho$ for both rocks were calculated as follows:

$$\frac{dk}{d\rho} \approx 2.0 \cdot 10^{12} \text{ c.g.s.} \quad \text{(for granite)}, \tag{11}$$

$$\frac{dk}{d\rho} \approx 2.2 \cdot 10^{12} \text{ c.g.s.} \quad \text{(for dunite)}. \tag{12}$$

From these values the values of dk/dP are calculated as follows:

$$\frac{dk}{dP} \approx 7 \quad \text{(for granite)}, \tag{13}$$

$$\frac{dk}{dP} \approx 4 \quad \text{(for dunite)}. \tag{14}$$

It ought to be remarked that these values may be altered by some twenty percent by applying different values of the thermal expansion coefficient of rock, the measured values of which are greatly diverse from each other even for rocks with the same name of granite or dunite.

From the above argument it will safely be said that the bulk modulus of rocks under a certain high pressure beyond the critical point may approximately be treated as that governed by an ordinary theory of perfect elasticity. Namely, the bulk modulus of rock in this case is considered to be derived mainly from the effect of interatomic or interionic action and partly from that caused by mosaic structure. From these reasons the relation of the bulk modulus of rock to its pressure will conveniently be compared with those for single crystalline metarials. Lazarus (**69**) measured the adiabatic bulk modulus of single crystal of alkali-halide under the pressure up to 10,000 bars and obtained the following relations:

$$\frac{dk}{dP} \approx 6 \quad \text{(for NaCl)}, \tag{15}$$

$$\frac{dk}{dP} \approx 8 \quad \text{(for KCl)}. \tag{16}$$

Bridgman (**70**) also measured the isothermal compressibility of single crystal of α-quartz up to the pressure of 100,000 bars and obtained the result that

$$\left(\frac{dk}{dP}\right)_{P=0} \approx 5 \quad \text{(for quartz)}. \tag{17}$$

The following value for the single crystal of MgO was calculated under the assumption that the bulk moduls depends only upon the density, as deduced from the data obtained by Durand (**71**) in his experiment on the temperature dependency of bulk modulus of MgO.

$$\frac{dk}{dP} \approx 4 \quad \text{(for MgO)}. \tag{18}$$

The above values of dk/dP are not influenced by the effect of mosaic structure

and they will be useful, to a certain degree, for the investigation of the bulk modulus of granite and dunite in which the contribution from the mosaic structure is considered to be partial. The mean value of dk/dP above obtained is

$$\frac{dk}{dP}=4\sim8\,. \tag{19}$$

In these cases the bulk modulus is derived only from the interionic action, and the obtained values are nearly of the same order of magnitude as those of granite and dunite. From these comparisons it will correctly, in the present approximation, be interpreted that the bulk moduli of granite and dunite in the pressure range beyond the critical point shown in Fig. 2 are mainly contributed by the interionic potential. In particular, as the bulk modulus of dunite is explained to be derived from the mutual action of Mg–O and Si–O, the accordance of the value of dk/dP of dunite with those obtained from the single crystals of MgO and quartz is favourable for the interpretation.

From the standpoint of the theory of solids, the interatomic potential ω can be assumed as:

$$\omega=\frac{A}{s^m}+\frac{B}{s^n}\,, \tag{20}$$

where s is the interatomic distance and A, B, m, n are certain constants proper to the material. Ramsay (**12**) obtained the following relation as deduced from the above potential:

$$\frac{dk}{dP}=\frac{1}{3}(m+n+6)\,. \tag{21}$$

For the case of dunite in which the mutual action of Mg–O, Si–O is assumed, the numerals are reasonably assigned as $m=1$ for Coulomb potential and $n=7$ after Born (**72**) for exchange force of Ne–nucleus in both cases of Mg and Si, and the calculated value is

$$\frac{dk}{dP}=4\sim5\,. \tag{22}$$

This value is nearly equal to that obtained for dunite. And on the other hand, it is to be emphasized that nearly the same value is also obtained from the finite strain theory of Murnaghan (**73**) as extended by Birch (**15**). From these considerations it is approximately concluded that the bulk modulus over the critical pressure shown in Fig. 2 is derived from the main contribution of the mutual action between ions and partially from the effect of the mosaic structure, and in this case the effects by hysteresis and temperature are negligibly small.

It is an important fact that both the bulk modulus and $dk/d\rho$ have a single relation with the density at the pressure of over 2,000 bars in the case of granite as seen

in Fig. 2a, and, on the other hand, in the case of dunite only the bulk modulus depends approximately on the density at the pressure of over 2,000 bars but not so for $dk/d\rho$. Namely, the asymptotic lines for the values of bulk modulus at various temperatures for dunite do not coverge so well to a common line even at the pressure of over 5,000 bars as shown in Fig. 2b. This tells us that the effect of pore space upon the values of bulk modulus and $dk/d\rho$ becomes negligibly small at the pressure of over 2,000 bars in the case of granite, but it persists at higher pressures in the case of dunite, especially for the value of $dk/d\rho$. And as the diminution of the pore space is caused by some processes of rupture, the effect above stated for every rock is intimately related with their own breaking strength. In simple calculation, the maximum stress difference at the boundary of a small spherical pore is roughly estimated as $3p_0/2$ when the hydrostatic pressure p_0 is exerted on the mother body (cf. Love (**66**)). From the calculation, the stress difference around the pore space is roughly estimated as 3,000 bars in the case of external pressure of 2,000 bars. The so-called breaking strength of ordinary rocks is less than 3,000 bars, but this strength represents a value of breaking strength at the weakest point and besides the exerting pressure in the measurement is not hydrostatical. As the shape of a pore space is actually never spherical, the above argument is not necessarily accurate quantitatively, but it may be concluded that it necessitates a very much higher stress difference than the breaking strength to crush the all pore spaces and eliminate its effect for the elasticity of rock. The breaking strength for granite is estimated to be generally lower than that for dunite, so that the pore space will be reduced greatly by a comparatively lower pressure in the case of granite, but in the case of dunite the higher pressure, perhaps up to 10,000 bars, will be required for reducing the pore space. As stated above, it is not only natural to examine the relation between the bulk modulus and density in order to eliminate the effect of temperature and porosity, but there is another advantage. Namely, as seen in Fig. 2, the density of dunite in this diagram covers a range four times as large as that in the $k\sim P$-diagram. And really the value of $dk/d\rho$, which is little influenced by the effect of porosity and temperature, is more accurately calculated from the $k\sim\rho$-diagram compared with that derived from the $k\sim P$-diagram, because the inclination $(dk/d\rho)$ of an asymptotic line to any curve in the $k\sim\rho$-diagram is uniquely determined under the condition of no intersection of the asymptote with the other curves.

Concerning the problem of temperature, the coefficient of thermal expansion is a subject of importance. From the values of the bulk modulus of dunite at various temperatures shown in Fig. 2b, it is experimentally concluded that the bulk modulus at a high temperature, in case of the same density, has a larger value than that at a low temperature. This means that the decreasing rate of thermal expansion coefficient with pressure is smaller than the increasing rate of bulk modulus with pressure. That

is, the product of the thermal expansion coefficient by bulk modulus is an increasing function of pressure. On the contrary, the coefficient of thermal expansion decreases with pressure more rapidly than does compressibility for ordinary substances, as shown by Bridgman (**62**). This tendency will be mainly due to porosity and this is the reason why it is more reasonable to determine dk/dP from the relation of density-bulk modulus than from the relation of pressure-bulk modulus.

In summarizing the present section, it may be concluded that, in the determination of the pressure coefficient (dk/dP) for the bulk modulus of rock in the range of high pressure of more than several thousands bars, the use of a $k\sim\rho$-diagram is more natural and accurate than the direct application of a $k\sim P$-diagram and that the value of dk/dP thus obtained from the data of rock-experiments is in a good agreement with that derived from the observed data of seismic waves.

3. Rigidity of rocks and the Earth's interior

As in case of the bulk modulus, the rigidity of rocks will greatly be influenced by the existence of micro-pore spaces, especially in the range of low external pressure. The same method, as in the case of the bulk modulus, was applied to the experimental rock data for the purpose of determining the pressure coefficient of rigidity of rocks ($d\mu/dP$). Namely, instead of the direct determination of $d\mu/dP$, the relation between the rigidity and density ($d\mu/d\rho$) was conveniently calculated, but in the present case it was not so successfully and accurately determined as in the case of the bulk modulus. The cause of failure may partly be attributable to the greater uncertainty in the determination of the shear wave velocity (V_s) compared with that of dilatational wave (V_p) in the case of rock-experiments, and partly to the stronger influences by temperature, viscosity, porosity, etc. upon the value and behaviour of the rigidity of rock compared with those upon the bulk modulus. From the modern theory of solids, the nature and state equation with regard to the rigidity is more complicatedly connected with the temperature, viscosity, and other complex quantities compared with those in the case of the bulk modulus, and for these reasons, the problem on the nature and behaviour of the rigidity of the material in the Earth's interior and their relation to the experimental data of rocks will be studied separately in a forthcoming paper from a special standpoint reasonably assumed.

In the present section some problems on the ratio of bulk modulus to rigidity and on the pressure coefficient of the dilatational wave velocity (dV_p/dP) will be treated. The ratio of bulk modulus to rigidity (k/μ) is simply derived from the values of velocities of both dilatational and shear waves as follows:

$$V_p=\sqrt{\frac{k+4\mu/3}{\rho}}, \quad V_s=\sqrt{\frac{\mu}{\rho}}, \tag{23}$$

$$\frac{k}{\mu}=\left(\frac{V_p}{V_s}\right)^2-\frac{4}{3}. \tag{24}$$

Thus, in the treatment of isotropic and perfect elasticity, the value of k/μ is simply derived from the ratio V_p/V_s and can be conveniently discussed without any knowledge of the density distribution of the Earth's interior. Practically, the value of k/μ in the range of the Earth's mantle is shown in Fig. 4 from the materials by H. Jeffreys (**54**). The curves were previously published in 1951 (**74**) by the writer and discussed in some detail. And on the other hand, the values of k/μ plotted against pressure, which were calculated from the experimental data by D. S. Hughes and J. H. Cross (**65**), are shown in Fig. 3.

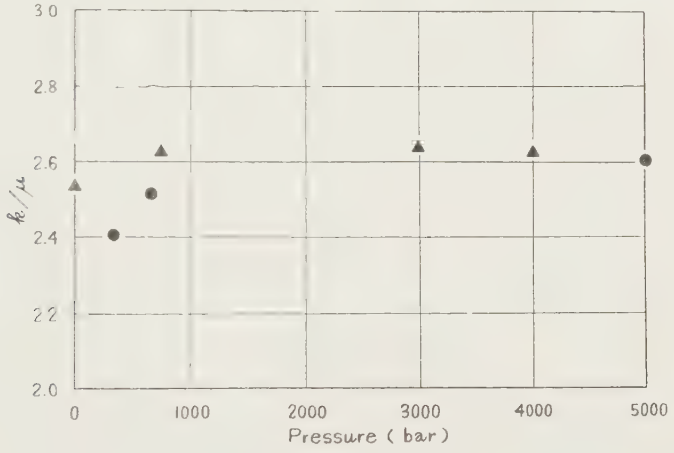

Fig. 3a. Relation between the value of k/μ of granite and the exerted pressure.
▲ (at 28°C), ● (at 200°C)

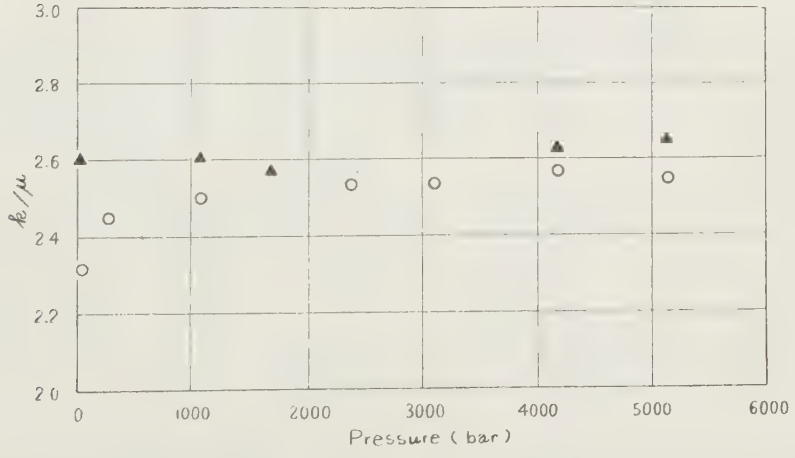

Fig. 3b. Relation between the value of k/μ of dunite and the exerted pressure.
▲ (at 24.5°C), ○ (at 300°C)

As seen in Fig. 3, the value of k/μ increases in the range of low pressure, but beyond the pressure of 2,000 bars, it becomes nearly constant or rather slightly decreases with the increase of pressure for both cases of granite and dunite. In the range of high pressure the values of k/μ determined from the rock-experiment are estimated to be nearly

$$\frac{k}{\mu} \approx 2.5 \qquad \text{(for granite)}, \tag{25}$$

$$\frac{k}{\mu} \approx 2.5 \qquad \text{(for dunite)}. \tag{26}$$

On the other hand, the value of k/μ obtained from Jeffreys' velocity distribution is

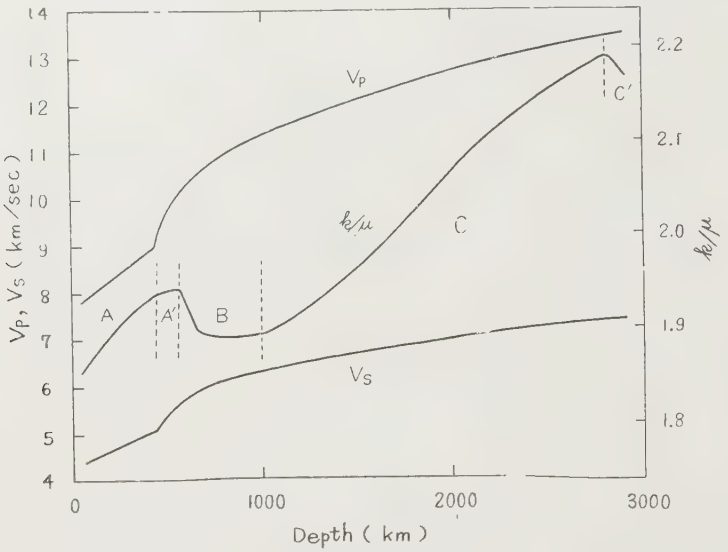

Fig. 4. Distribution of the values of V_P, V_S, and k/μ in the Earth's mantle.

estimated to be nearly 1.8 at the mantle surface as shown in Fig. 4. The inconsistency between the values of k/μ obtained from the rock-experiment and seismic observation is considered to be caused from various sources and its solution will offer an important key towards clarifying the problem on the character of rigidity distribution of the Earth's interior and especially the existence of a central core of zero rigidity. Recently P. W. Bridgman (**75**) has found a large increase in rigidity at pressures of about 85,000 atm. in the course of rock-experiments on dunite. The pressure of 85,000 atm. corresponds to that at the depth of about 250 km in the Earth, and the so-called 13°-discontinuity is also estimated to lie at that depth in the Earth's mantle. From the reasons above described and other reasons the problem on the character of rigidity in the Earth's interior is considered to be an important and essential item of research, and it will be treated separately in a near future in a forthcoming article.

In addition, the pressure coefficient of V_p will be discussed to some extent in the following, putting aside the problem of V_s. In Fig. 5, the relation between V_p (the dilatational wave velocity) and ρ (density) which are derived from Hughes' rock-experiment previously mentioned (**65**). This figure corresponds to Fig. 2 which represents the relation between bulk modulus and density. All curves at different temperatures converge to an asymtotic line which is approximately represented by a line connecting the points at 5,000 bars. This fact is reasonably interpreted, as in case of the bulk modulus, from the effect that the repulsive forces between the con-

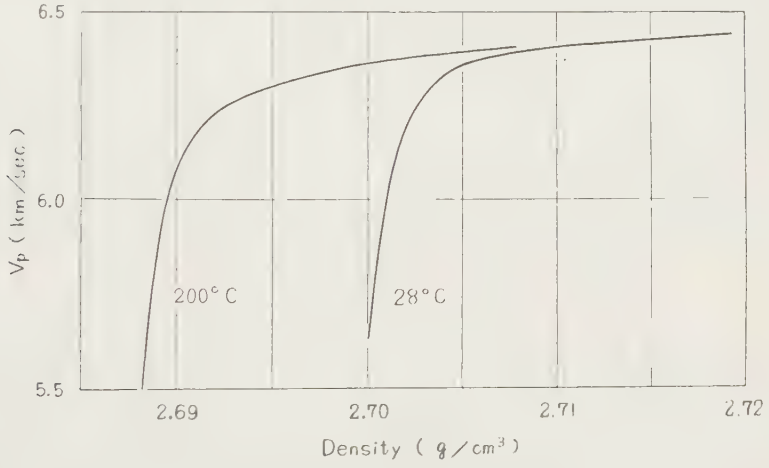

Fig. 5a. Relation between the dilatational wave velocity of granite and the density.

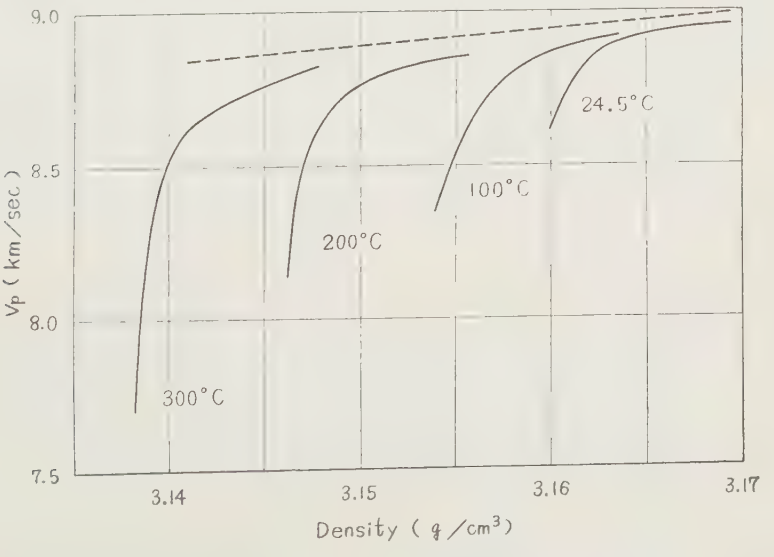

Fig. 5b. Relation between the dilatational wave velocity of dunite and the density.

stituent ions in the rocks become more and more predominant compared with the effects of porosity and temperature. From the gradient of the asymptote, we obtain:

$$\frac{dV_p}{d\rho} \approx 5 \text{ km/sec. g/cm}^3 \quad \text{(for granite)}, \tag{27}$$

$$\frac{dV_p}{d\rho} \approx 6 \text{ km/sec. g/cm}^3 \quad \text{(for dunite)}, \tag{28}$$

which correspond respectively to

$$\frac{dV_p}{dP} \approx 0.2 \text{ km/sec. } 10^4 \text{ bars} \quad \text{(for granite)}, \tag{29}$$

$$\frac{dV_p}{dP} \approx 0.1 \text{ km/sec. } 10^4 \text{ bars} \quad \text{(for dunite)}. \tag{30}$$

It ought to be remarked that the values above obtained may be altered by somewhat 20 percent, owing to the uncertainty of the thermal expansion coefficients, as in case of the bulk modulus. On the other hand, from the observed velocity of seismic waves by H. Jeffreys and by B. Gutenberg, the following value is estimated at the depth of 300 km:

$$\frac{dV_p}{dP} \approx 0.1 \text{ km/sec. } 10^4 \text{ bars}. \tag{31}$$

The value is in a good agreement with that obtained in (30). Accordingly the pressure coefficient of the velocity of dilatational seismic wave is correctly determined from the data of rock-experiments by the process of calculating first the value of $dV_p/d\rho$ and then converting it to the value of dV_p/dP, as in case of the determination of pressure coefficient of bulk modulus. In both cases, the influences of temperature, porosity, viscosity, etc. upon the determination of the pressure coefficient of the bulk modulus and the dilatational wave velocity of rocks are considered to be favourably minimized by the above-described method of analysis. In fact, the value of dV_p/dP obtained from Fig. 1b is twice larger than that obtained from Fig. 5b for dunite, and for the case of granite, the values obtained from Fig. 1a and Fig. 5a are in a good agreement.

4. Summary

The change with pressure of elasticity of rock obtained from laboratory experiments was discussed with reference to those derived from the analysis of velocity distribution of seismic waves in the Earth's interior. Putting aside the problem of the rigidity of rocks, the pressure coefficient of the bulk modulus and the dilatational wave velocity, which were determined from the data of rock-experiment by the present process of treatment of the data on the bulk modulus and dilatational wave velocity of rock with respect to the density instead of the pressure itself, were ascertained to

be in a fairly good agreement with the values derived from the observation of seismic waves. It can safely be said that the various effects caused by temperature, porosity, viscosity, etc. upon the characters of rock-elasticity were minimized in the present method of analysis, and in this way, the marked discrepancy with respect to the pressure coefficient of elasticity hitherto discussed, between the values obtained from the rock-experiment and the values obtained from the seismic waves in the Earth's interior was certainly reconciled. On the other hand, the character of rigidity is considered to be very complicated compared with that of bulk modulus. Since its investigation is certainly important for the solution of many essential problems of the Earth's interior, the problem on rigidity will be discussed separately in a forthcoming paper.

Acknowledgments

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